The art of calorimetry part III

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DESY

Measurement of showers

or "from signal back to energy"

To make a statement about the energy of a particle:

1. relationship between measured signal and deposited energy

Detector response ➔ Linearity

- The average calorimeter signal vs. the energy of the particle
- Homogenous and sampling calorimeters
- Compensation

2. precision with which the unknown energy can be measured

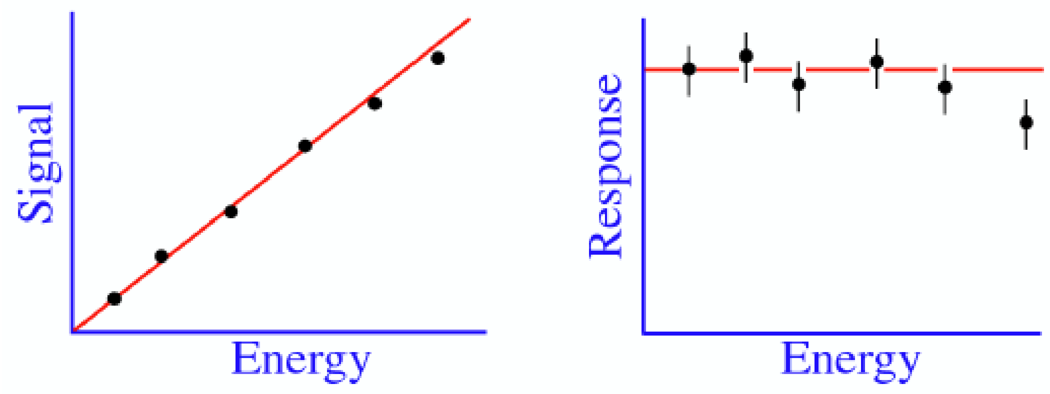
Detector resolution ➔ Fluctuations

- Event to event variations of the signal
- Resolution
  - What limits the accuracy at different energies?

Response and linearity

"response = average signal per unit of deposited energy"
e.g. # photoelectrons/GeV, picoCoulombs/MeV, etc

A linear calorimeter has a constant response



In general

- Electromagnetic calorimeters are linear
  ➔ All energy deposited through ionization/excitation of absorber
- Hadronic calorimeters are not

Sources of non-linearity

Instrumental effects

- Saturation of gas detectors, scintillators, photo-detectors, electronics

Response varies with something that varies with energy

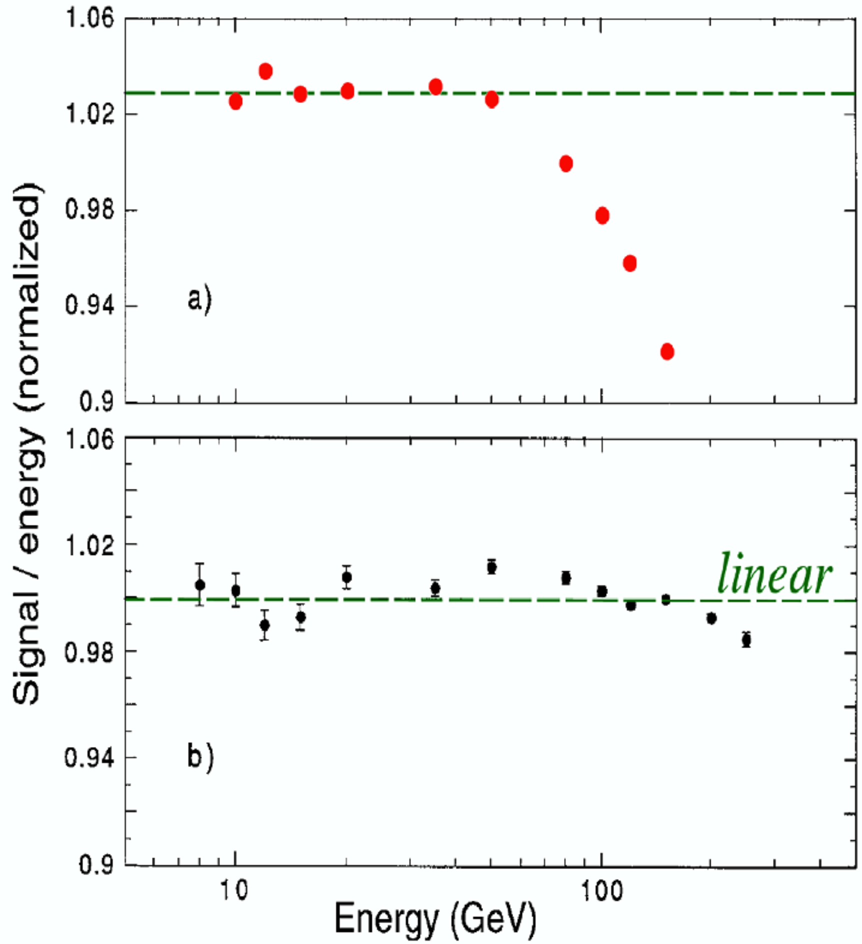
Examples:

- Deposited energy “counts” differently, depending on depth
  - And depth increases with energy
- Electromagnetic and hadronic energies “count” differently
  - And EM fraction increases with energy

Leakage (increases with energy)

Example of non-linearity

Signal linearity for electromagnetic showers



Before

After correction of PMT response

FIG. 3.1. The em calorimeter response as a function of energy, measured with the QFCAL calorimeter, before (*a*) and after (*b*) precautions were taken against PMT saturation effects. Data from [Akc 97].

Homogenous calorimeters

One block of material serves as absorber and active medium at the same time

- Scintillating crystals with high density and high Z

Advantages:

- see all charged particles in the shower ➔ best statistical precision
- same response from everywhere ➔ good linearity

Disadvantages:

- cost and limited segmentation

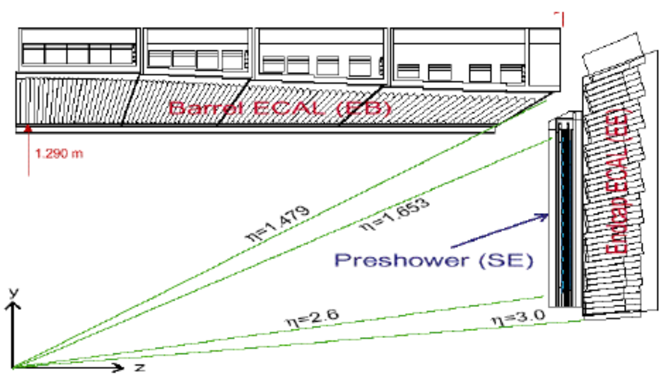
Examples:

- B factories: small photon energies
- CMS ECAL:

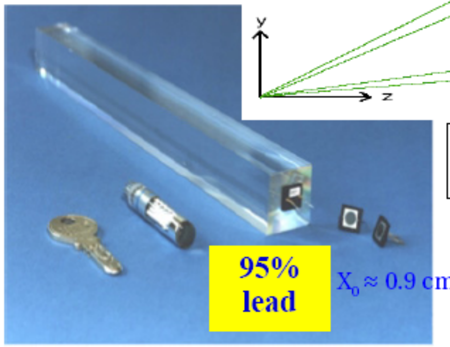
  optimized for H→γγ

CMS ECAL

CMS $PbWO_4$ crystal calorimeter



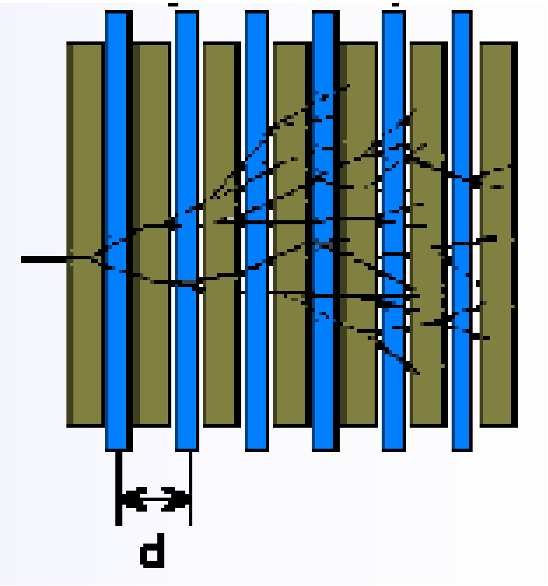
- Barrel: 62k crystals 2.2 × 2.2 × 23 cm
- End-caps: 15k crystals 3 × 3 × 22 cm



Sampling calorimeters

Use different media

- High density absorber
- Interleaved with active readout devices
- Most commonly used: sandwich structures ➔
- But also: embedded fibres, ….



Sampling fraction

- $f_{sampl} = E_{visible} / E_{total\ deposited}$

Advantages:

- Cost, transverse and longitudinal segmentation

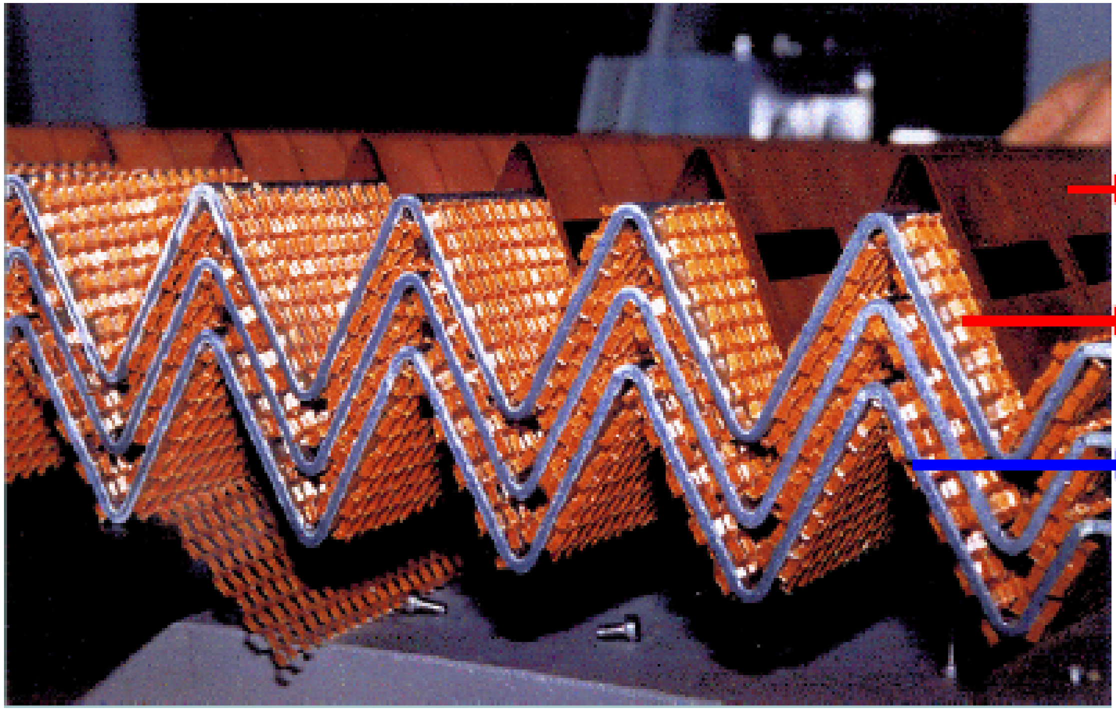
Disadvantages:

- Only part of shower seen, less precise

Examples:

- ATLAS ECAL
- All HCALs (I know of)

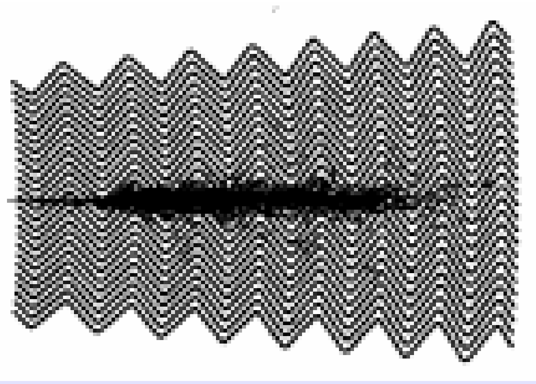
ATLAS LAr ECAL

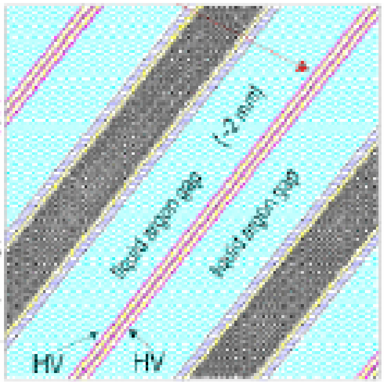


Cu electrodes at +HV

Spacers define LAr gap
2×2 mm

2 mm Pb absorber
clad in stainless steel.





Sampling calorimeters

Sampling fractions f_{sampl} are usually determined with a mip (minimum *dE/dx*)

NB. mip do not exist !

e.g. D0 EM section:

3mm ^{238}U	*dE/dx* = 61.5 MeV/layer	f_{sampl} = 13.7%
2x2.3mm LAr	*dE/dx* = 9.8 MeV/layer	

However, for EM showers, the sampling fraction is only 8.2% ➔ e/mip ~ 0.6

- e/mip is a function of the shower depth, in U/LAr it decreases

  e/mip increases when the sampling frequency becomes very high

This is because ➔ Photoelectric effect: $\sigma \propto Z^5$, $(18/92)^5 \sim 3\ 10^{-4}$

➔ Soft γs (E<1MeV) are very inefficiently sampled

- Effect strongest at high Z and late in the shower development
- Typical range for photoelectrons < 1mm
- Only photoelectrons produced near the boundary between active and passive material produce a signal

Important !!! watch the MC cutoff scale

→if absorber layer are thin, they may contribute to the signal

Sampling calorimeters: e/mip

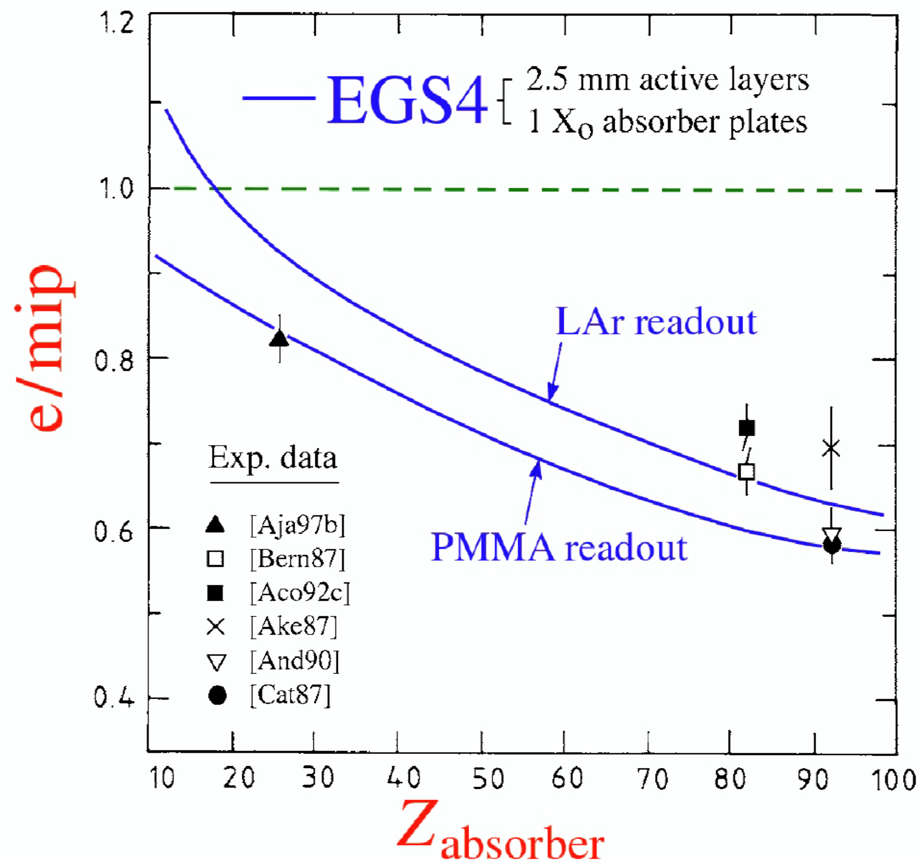


FIG. 3.7. The e/mip ratio for sampling calorimeters as a function of the Z value of the absorber material, for calorimeters with plastic scintillator or liquid argon as active material. Experimental data are compared with results of EGS4 Monte Carlo simulations [Wig 87].

- e/mip larger for LAr (Z=18) than for scintillator (Z~6-7)
- e/mip ratio determined by the difference in Z values between active and passive media

PMMA=polymethylethacrylate=
Plastic scintillator

e/mip dependence of shower depth

The EM sampling fraction changes with depth!

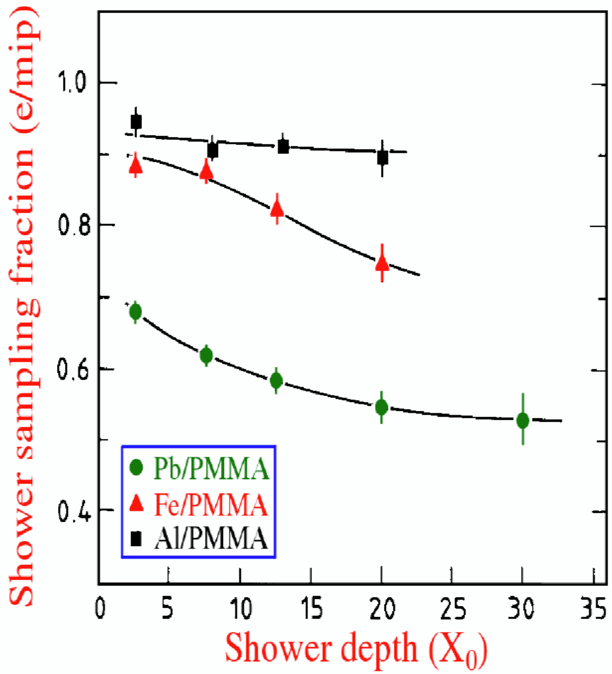


FIG. 3.8. The e/mip ratio as a function of the shower depth, or age, for 1 GeV electrons in various sampling calorimeter configurations. All calorimeters consist of 1 X_0 thick absorber layers, interleaved with 2.5 mm thick PMMA layers. Results from EGS4 Monte Carlo simulations [Wig 87].

e/mip changes as the shower develops
The effect can be understood from the changing composition of the showers

- Early phase: relatively fast shower particles (pairs)
- Tails dominated by Compton and photoelectric electrons

Relevant for longitudinally segmented ECAL: must use different calibration constants

EM and hadronic response

The response to the hadronic part (h) of a hadron-induced shower is usually smaller than that to the electromagnetic part (e)

- Due to the invisible energy
- Due to short range of spallation nucleons
- Due to saturation effects for slow, highly ionizing particles

➔If a calorimeter is linear for electrons, it is non-linear for hadrons

The condition e = h is known as COMPENSATION

➔can be obtained in non-homogeneous calorimeters with proper choice of materials/ material thickness

Homogeneous calorimeters are in general non-compensating (h/e < 1)

➔ response to hadron showers smaller than to the electromagnetic one

but, because of similarity between the energy deposit mechanism response to muons and em showers are equal

$\Rightarrow$ same calibration constant $\Rightarrow$ e/mip=1

e/h and e/π, (non-) linearity

e/h: not directly measurable ➔ give the degree of non-compensation

e/π: ratio of response between electron-induced and pion-induced shower

$$\frac{e}{\pi} = \frac{e}{f_{em} e + (1 - f_{em})\, h} = \frac{e}{h} \cdot \frac{1}{1 + f_{em}\,(e/h - 1)}$$

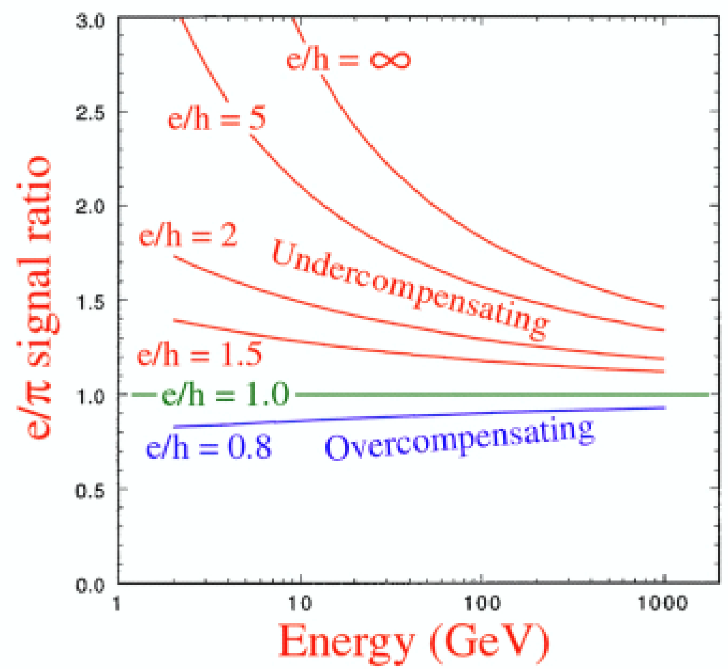
e/h is energy independent

e/π depends on E via $f_{em}(E)$ ➔ non-linearity

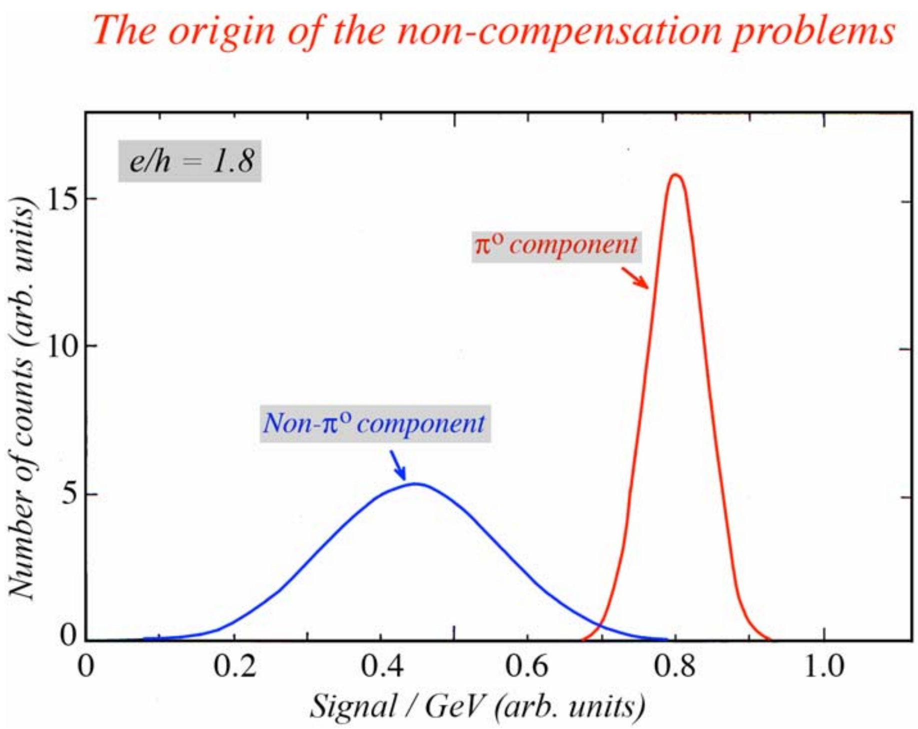
Approaches to achieve compensation:

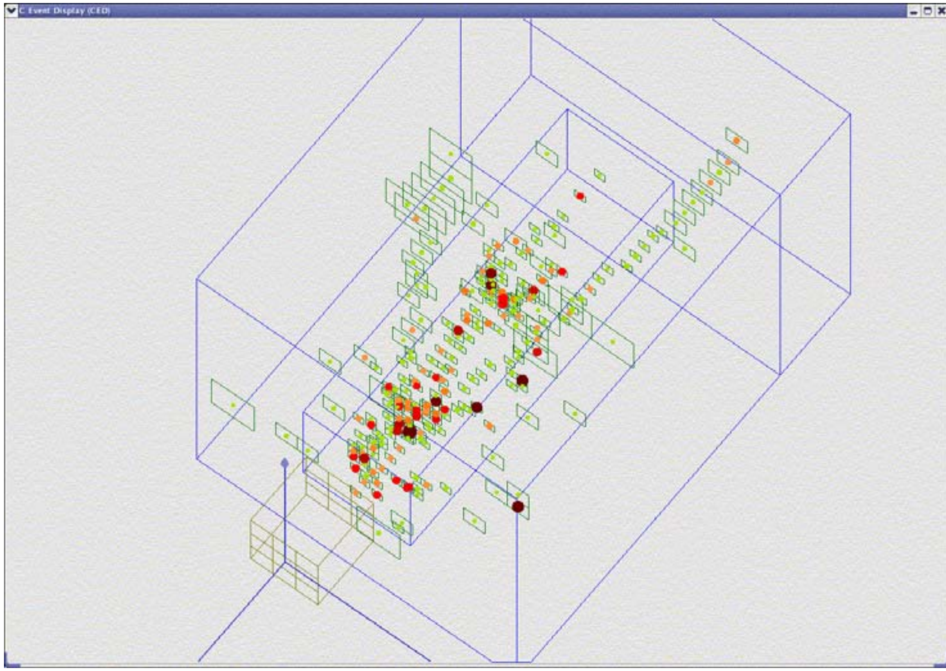
e/h → 1 right choice of materials or

f_{em} → 1 (high energy limit)



EM fraction





Charge conversion of $\pi^{+/-}$ produces electromagnetic component of hadronic shower (π^0)

20 GeV pion shower in a Scint.-Fe calorimeter

High energetic EM "clusters" visible

Energy dependence of EM component

$f_{em} \rightarrow 1$ (high energy limit)

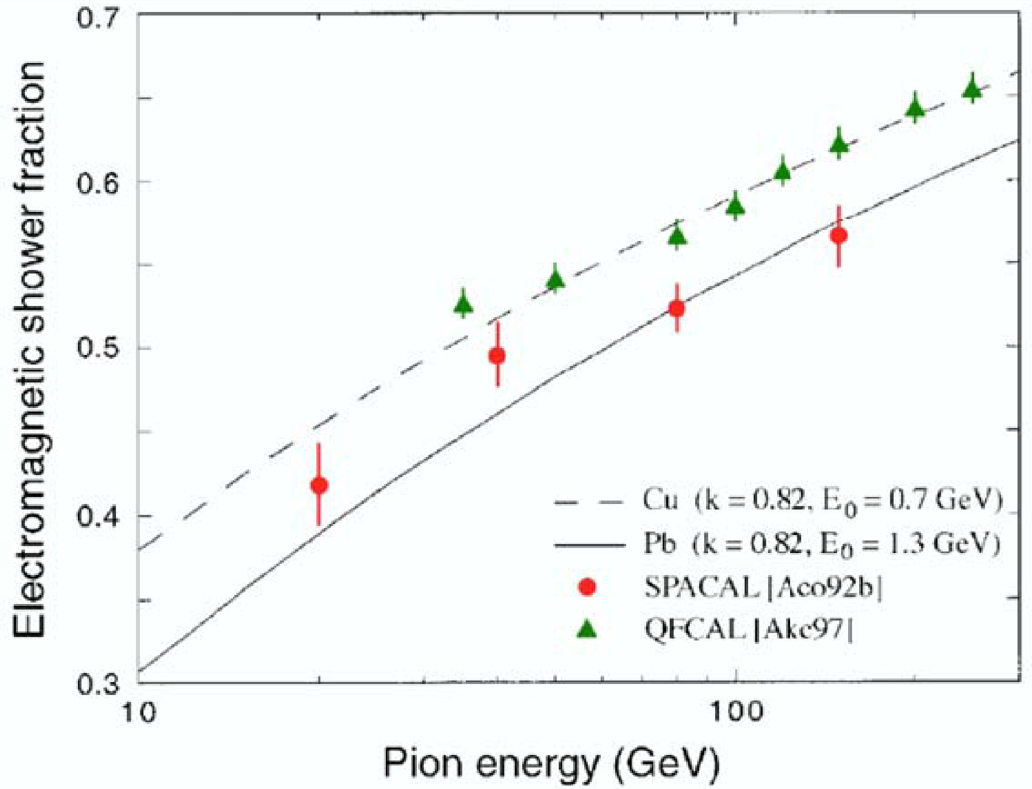


FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

Hadron non-linearity and e/h

Non-linearity determined by e/h value of the calorimeter

Measurement of non-linearity is one of the methods to determine e/h

- Assuming linearity for EM showers, $e(E_1)=e(E_2)$:

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1)+[1-f_{em}(E_1)]\cdot e/h}{f_{em}(E_2)+[1-f_{em}(E_2)]\cdot e/h}$$

For e/h=1 ➔ $$\frac{\pi(E_1)}{\pi(E_2)} = 1$$

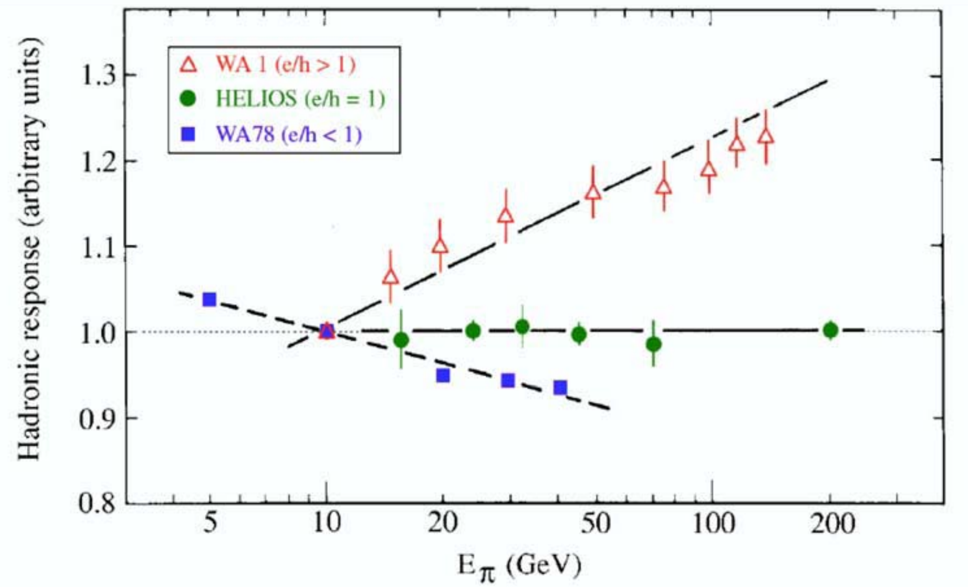


FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter ($e/h > 1$, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter ($e/h < 1$, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.

Hadronic response (I)

Energy deposition mechanisms relevant for the absorption of the non-EM shower energy:

- Ionization by charged pions f_{rel} (Relativistic shower component).
- spallation protons f_p (non-relativistic shower component).
- Kinetic energy carried by evaporation neutrons f_n
- The energy used to release protons and neutrons from calorimeter nuclei, and the kinetic energy carried by recoil nuclei do not lead to a calorimeter signal. This is the invisible fraction f_{inv} of the non-em shower energy

The total hadron response can be expressed as:

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$

$$f_{rel} + f_p + f_n + f_{inv} = 1$$

Normalizing to mip and ignoring (for now) the invisible component

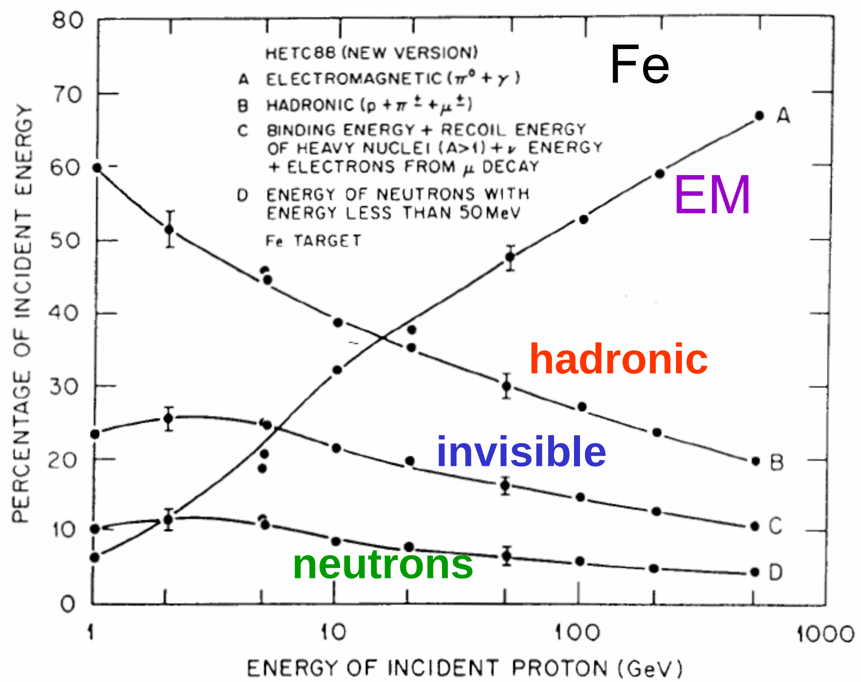
$$\frac{e}{h} = \frac{e/mip}{f_{rel} \cdot rel/mip + f_p \cdot p/mip + f_n \cdot n/mip}$$

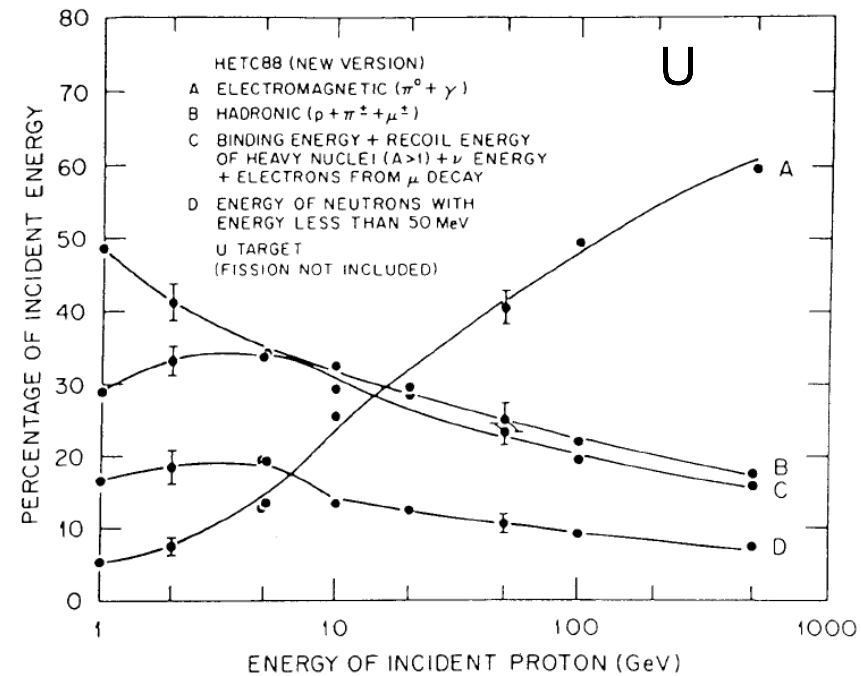
The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower

Hadronic shower: energy fractions

$$E_p = f_{em} e + (1 - f_{em}) h$$

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$





Hadronic response (II)

$$\frac{e}{h} = \frac{e/mip}{f_{rel} \cdot rel/mip + f_p \cdot p/mip + f_n \cdot n/mip}$$

The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower

Need to understand response to typical shower particles (relative to mip)

1. Relativistic charged hadrons

Even if relativistic, these particles resemble mip in their ionization losses

$\Rightarrow$ rel/mip = 1

Hadronic response (II)

$$\frac{e}{h} = \frac{e/mip}{f_{rel} \cdot rel/mip + f_p \cdot p/mip + f_n \cdot n/mip}$$

The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower

Need to understand response to typical shower particles (relative to mip)

2. Spallation protons
   More efficient sampling (p/mip>1)
   Signal saturation

Spallation protons

Aspects of compensation: Sampling of soft shower protons

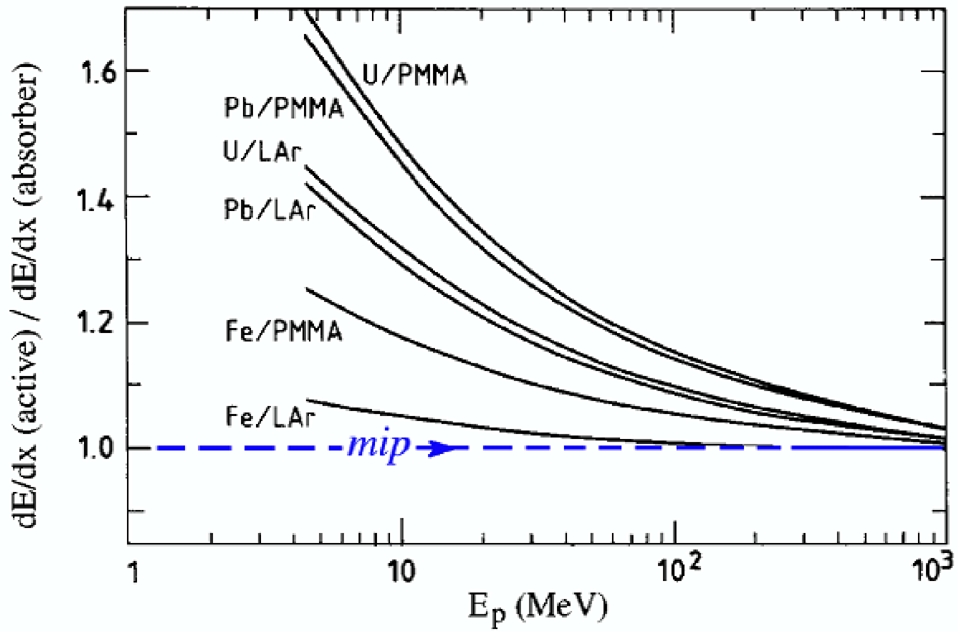


FIG. 3.15. The ratio of energy deposition by non-relativistic protons in the active and passive materials of various calorimeter structures, as a function of the proton s kinetic energy. This ratio is normalized to the one for mips. From [Wig 87].

- More efficient sampling (p/mip>1)

Spallation protons

Aspects of compensation: Saturation effects

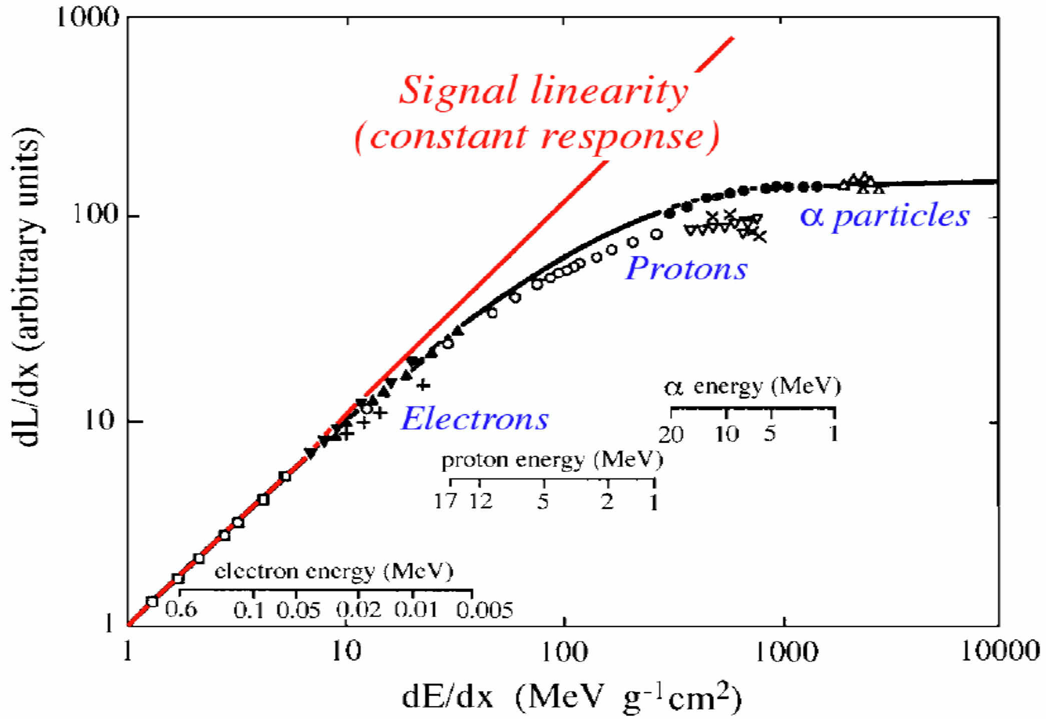


FIG. 3.25. Variation of the specific fluorescence, dL/dx, with the specific ionization loss, dE/dx, in anthracene crystals. The solid curve represents Equation 3.13 with $k_B = 6.6$ mg cm^{-2} MeV^{-1}.

- Signal saturation

Birk's law:

$$\frac{dL}{dx} = S\frac{dE/dx}{1 + k_b \cdot dE/dx}$$

Hadronic response (II)

$$\frac{e}{h} = \frac{e/mip}{f_{rel} \cdot rel/mip + f_p \cdot p/mip + f_n \cdot n/mip}$$

The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower

Need to understand response to typical shower particles (relative to mip)

3. Evaporation neutrons
   - $(n, n'\gamma)$ inelastic scattering: not very important
   - (n, n') elastic scattering: most interesting
   - (n, γ) capture (thermal): lots of energy, but process is slow (μs)

The role of neutrons

- Elastic scattering $f_{elastic} = 2A/(A+1)^2$
  - Hydrogen $f_{elastic} = 0.5$ / Lead $f_{elastic} = 0.005$
  - Pb/H_2 calorimeter structure (50/50)
  - 1 MeV n deposits 98% in H_2
  - mip deposits 2.2% in H_2

  } n/mip = 45

- Recoil protons can be measured!

⇒ Neutrons have an enormous potential to amplify hadronic shower signals, and thus compensate for losses in invisible energy

- Tune the e/h value through the sampling fraction!
  - e.g. 90% Pb/10% H_2 calorimeter structure
  - 1 MeV n deposits 86.6% in H_2
  - mip deposits 0.25% in H_2

  } n/mip = 350

Compensation by tuning neutron response

Compensation with hydrogenous active detector
Elastic scattering of soft neurons on protons

- High energy transfer
- Outgoing soft protons have high specific energy loss

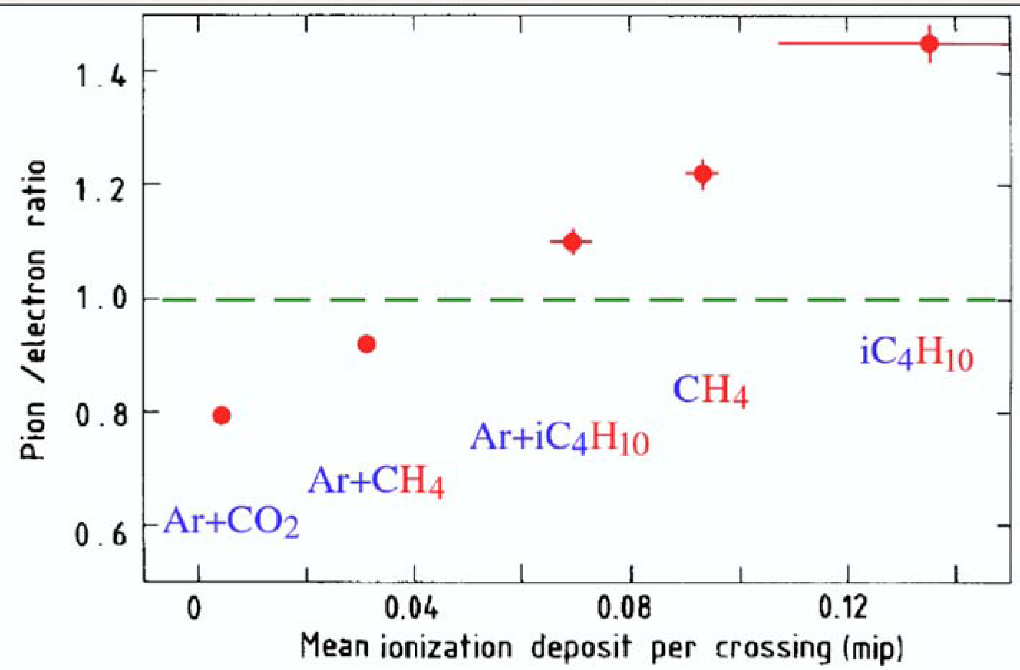


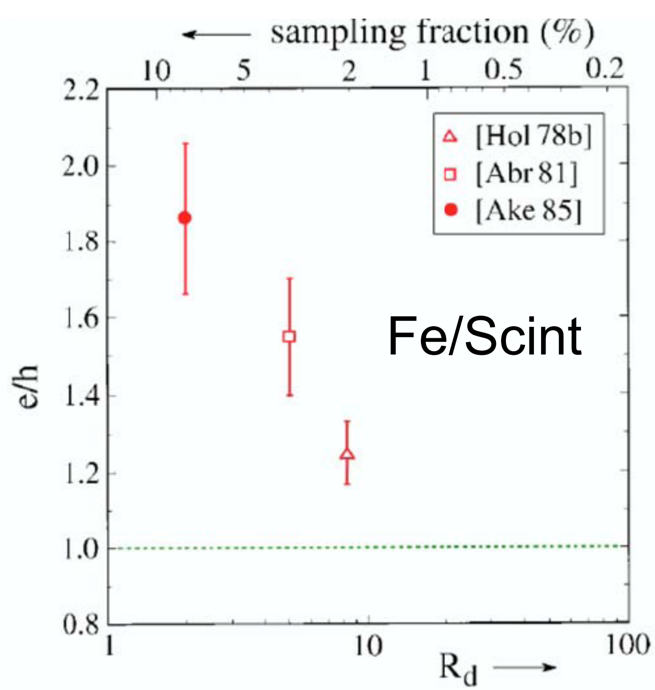
FIG. 3.32. The pion/electron signal ratio, averaged over the energy range 1 5 GeV, measured for different gas mixtures with the uranium/gas calorimeter of the L3 Collaboration. The horizontal scale gives the (calculated) average energy deposit in a chamber gap by slow neutrons [Gal 86].

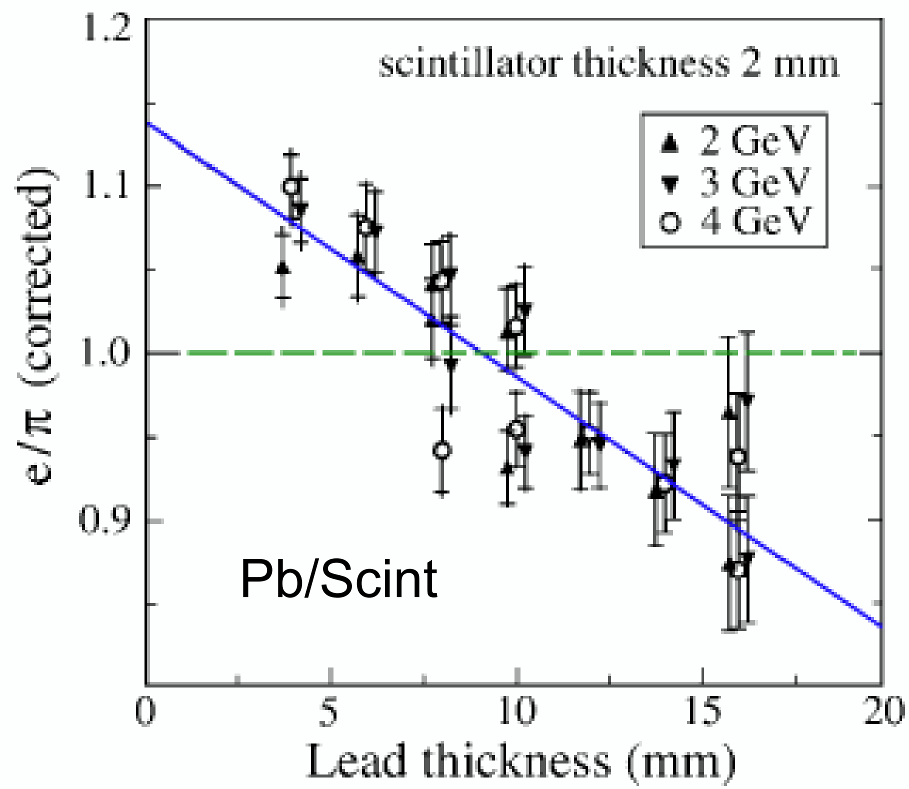
Compensation by tuning neutron response

Compensation adjusting the sampling frequency

Works best with Pb and U

In principle also possible with Fe, but only few n generated





the ratio 4:1 gives compensation for Pb/Scint

in Fe/Scint need ratio > 10:1 ➔ deterioration of longitudinal segmentation

Energy released by slow neutrons

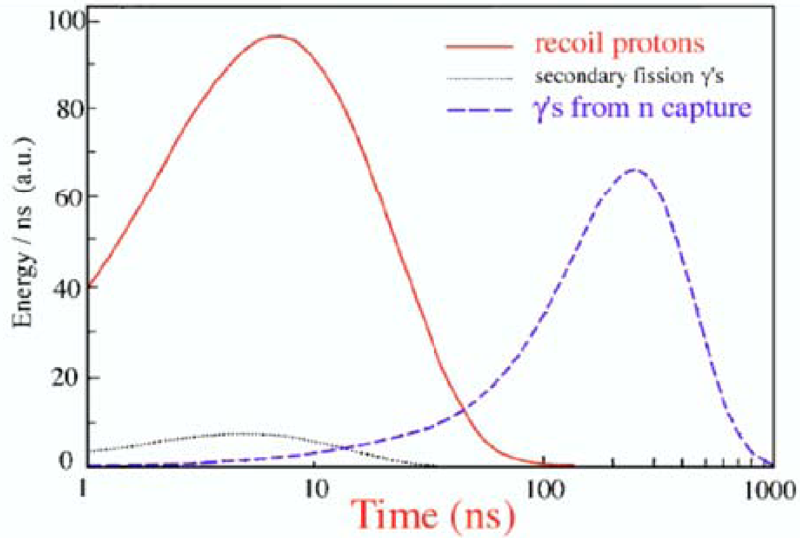


FIG. 3.22. Time structure of various contributions from neutron-induced processes to the hadronic signals of the ZEUS uranium/plastic-scintillator calorimeter [Bru 88].

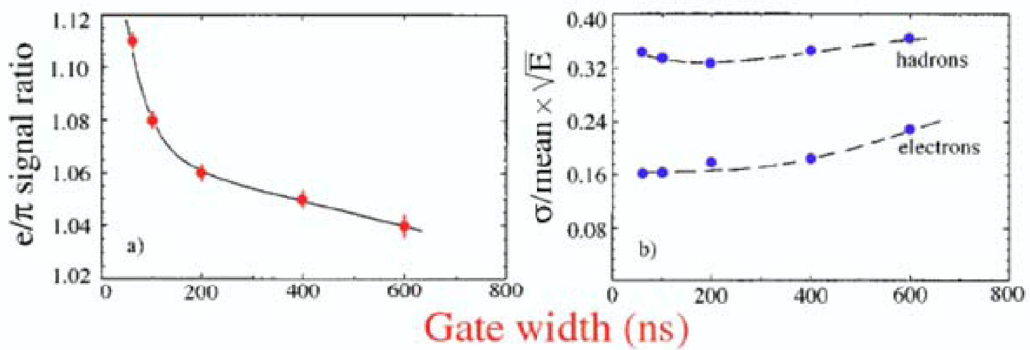


FIG. 3.23. The ratio of the average ZEUS calorimeter signals from 5 GeV/c electrons and pions (a) and the energy resolutions for detecting these particles (b), as a function of the charge integration time [Kru 92].

Large fraction of neutron energy captured and released after >100ns

Long integration time:
- collect more hadron E
➔ closer to compensation
- integrate additional noise
➔ worse resolution

Compensation: e/h=1

Hardware compensation ➔

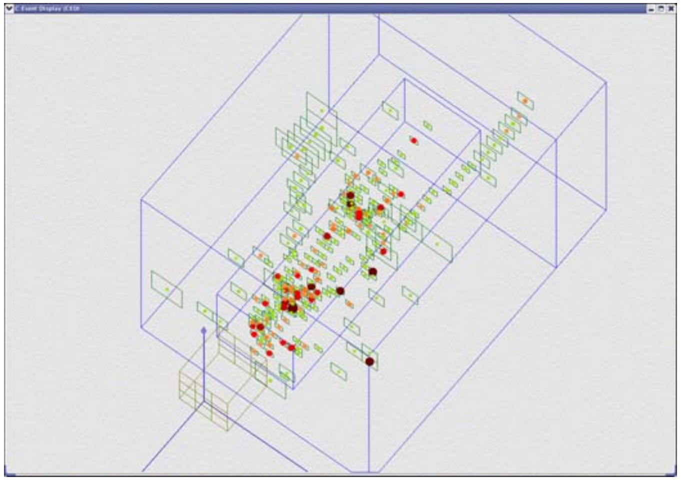
- Reduce EM response
  - High Z, soft photons
- Increase hadronic response
  - Ionization part
  - Neutron part (correlated with binding energy loss)

sampling calorimeter

- hydrogenous active medium (recoil p)
- precisely tuned sampling fraction
e.g. 10% for U/scint, 3% for Pb/scint

Software compensation

- Identify EM hot spots and down-weight
  - Requires high 3D segmentation



Hardware + Software compensation

- Measure EM component of shower
- Use measurement to re-weight hadron E

➔ Dual readout calorimeter

Summary on calorimeter response

To make a statement about the energy of a particle:

1. relationship between measured signal and deposited energy (response = average signal per unit of deposited energy)

Electromagnetic calorimeters have a linear response

➔ All energy deposited through ionization/excitation of absorber

Hadronic calorimeters are non-linear

➔ linear for electrons, non-linear for hadrons when $e/h \neq 1$

Compensation & the role of neutrons in hardware compensation

Next: - software compensation = the role of high granularity
- energy weighting

2. energy resolution (precision with which the unknown energy can be measured)

➔ let's talk about fluctuations

Measurement of showers

or “from signal back to energy”

Detector response ➔ Linearity

- The average calorimeter signal vs. the energy of the particle
- Homogenous and sampling calorimeters
- Compensation

Detector resolution ➔ Fluctuations

- Event to event variations of the signal
- Resolution
    - What limits the accuracy at different energies?

Fluctuations

Calorimeter's energy resolution is determined by *fluctuations* in the processes through which the energy is degraded (unavoidable)

- ultimate limit to the energy resolution in em showers (worsened by detection techniques)
- not a limit for hadronic showers ? (clever readout techniques can allow to obtain resolutions better than the limits set by internal fluctuations

➔ applying overall weighting factors (offline compensation) has no merit in this context

Many sources of fluctuations may play a role, for example:

- Signal **quantum** fluctuations (e.g. photoelectron statistics)
- **Sampling** fluctuations
- Shower **leakage**
- **Instrumental** effects (e.g. electronic noise, light attenuation, non-uniformity)

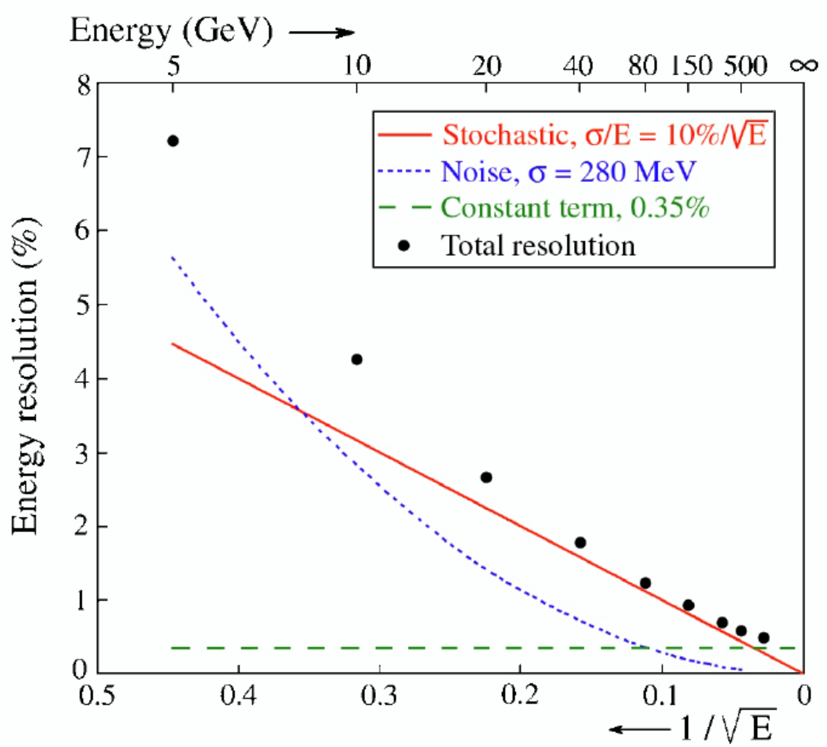
Fluctuations

Different effects have different energy dependence

- quantum, sampling fluctuations $\sigma/E \sim E^{-1/2}$
- shower leakage $\sigma/E \sim E^{-1/4}$
- electronic noise $\sigma/E \sim E^{-1}$
- structural non-uniformities $\sigma/E = \text{constant}$

Add in quadrature:

$$\sigma^2_{tot} = \sigma^2_1 + \sigma^2_2 + \sigma^2_3 + \sigma^2_4 + \ldots$$



← example: ATLAS EM calorimeter

Energy resolution

Ideally, if all shower particles counted: $E \sim N, \quad \sigma \sim \sqrt{N} \sim \sqrt{E}$

In practice:

absolute $\sigma = a\sqrt{E} \oplus b\,E \oplus c$

relative $\sigma / E = a / \sqrt{E} \oplus b \oplus c / E$

a: stochastic term

- intrinsic statistical shower fluctuations
- sampling fluctuations
- signal quantum fluctuations (e.g. photo-electron statistics)

b: constant term

- inhomogeneities (hardware or calibration)
- imperfections in calorimeter construction (dimensional variations, etc.)
- non-linearity of readout electronics
- fluctuations in longitudinal energy containment (leakage can also be $\sim E^{-1/4}$)
- fluctuations in energy lost in dead material before or within the calorimeter

c: noise term

- readout electronic noise
- Radio-activity, pile-up fluctuations

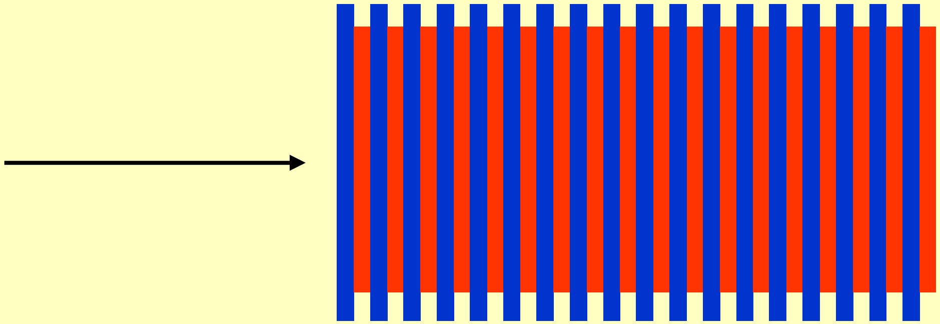
Don't forget the threshold!

Calorimeter types

There are two general classes of calorimeter:

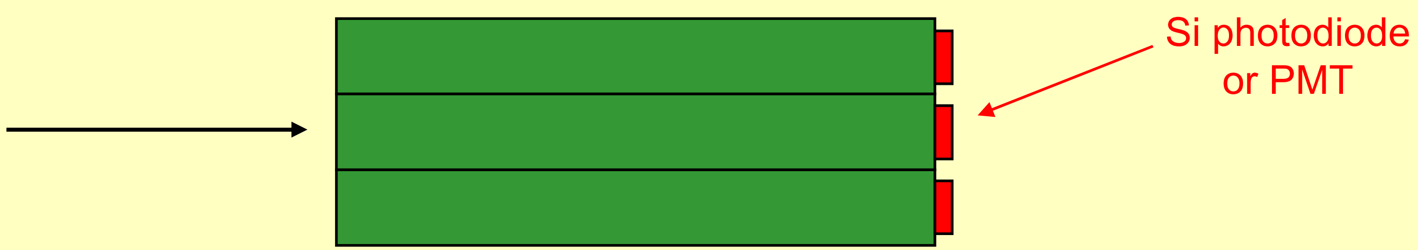
Sampling calorimeters:

Layers of passive absorber (such as Pb, or Cu) alternate with active detector layers such as Si, scintillator or liquid argon



Homogeneous calorimeters:

A single medium serves as both absorber and detector, eg: liquified Xe or Kr, dense crystal scintillators (BGO, $PbWO_4$ …….), lead loaded glass.



Intrinsic Energy Resolution of EM calorimeters

Homogeneous calorimeters:

signal amplitude $\propto$ sum of all E deposited by charged particles with $E>E_{threshold}$

If *W* is the mean energy required to produce a 'signal quantum' (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal) ➔ mean number of 'quanta' produced is $\langle n \rangle = E / W$

The intrinsic energy resolution is given by the fluctuations on n.

$$\sigma_E / E = 1/\sqrt{n} = \sqrt{(E/W)}$$

i.e. in a semiconductor crystals (Ge, Ge(Li), Si(Li))

W = 2.9 eV (to produce e-hole pair)

➔ 1 MeV γ = 350000 electrons ➔ $1/\sqrt{n}$ = 0.17% stochastic term

In addition, fluctuations on n are reduced by correlation in the production of consecutive e-hole pairs: the Fano factor F

$$\sigma_E / E = \sqrt{(FL/T)} = \sqrt{(FW/E)}$$

For GeLi γ detector $F \sim 0.1$ ➔ stochastic term $\sim 1.7\% / \sqrt{E[GeV]}$

Resolution of crystal EM calorimeters

Study the example of CMS: PbWO**4** crystals r/o via APD:

Fano factor *F ~ 2* for the crystal/APD combination
in crystals *F ~ 1* + fluctuations in the avalanche multiplication process of APD ('excess noise factor')

$PbWO_4$ is a relatively weak scintillator. In CMS, ~ 4500 photo-electrons/1 GeV (with QE ~80% for APD)

Thus, expected stochastic term:

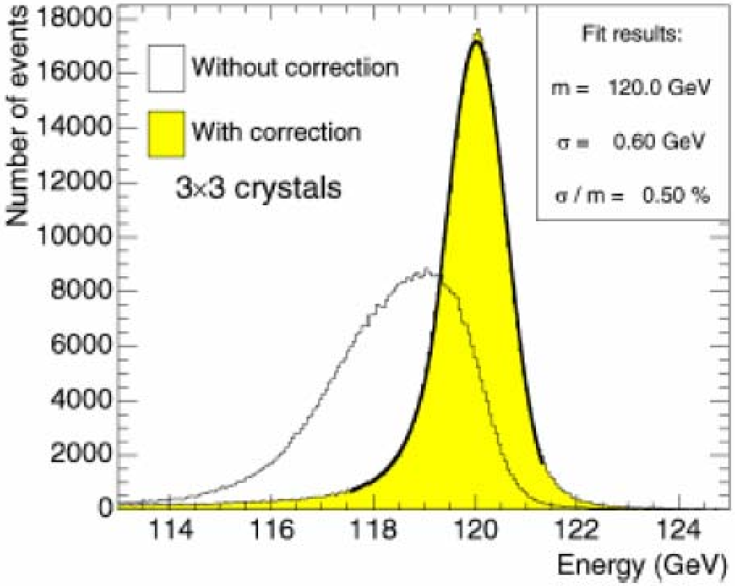
$$a_{pe} = \sqrt{(F/N_{pe})} = \sqrt{(2/4500)} = 2.1\%$$

Including effect of lateral leakage from limited clusters of crystals (to minimise electronic noise and pile up) one has to add

$$a_{leak} = 1.5\% \ (\Sigma(5x5)) \quad \text{and} \quad a_{leak} = 2\% \ (\Sigma(3x3))$$

Thus for the $\Sigma(3x3)$ case one expects $a = a_{pe} \oplus a_{leak} = 2.9\%$
➔ compared with the measured value: $a_{meas} = 3.4\%$

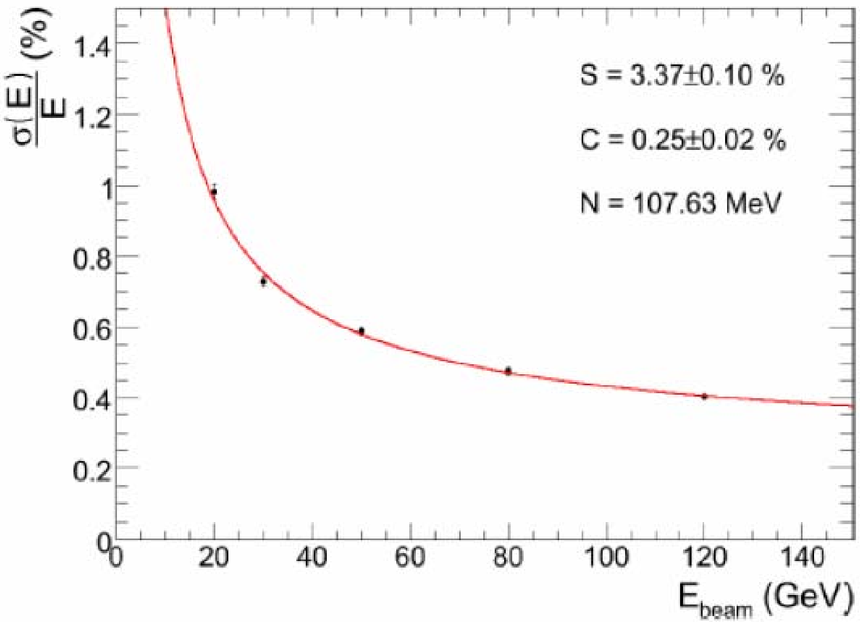
Example: CMS ECAL resolution



Correction for radial loss

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{3.37\%}{\sqrt{E}}\right)^2 + \left(\frac{0.107}{E}\right)^2 + (0.25\%)^2$$

stoch. noise const.



The sampling term is 3 times smaller than ATLAS; other terms are similar

Resolution of sampling calorimeters

Main contribution: sampling fluctuations, from variations in the number of charged particles crossing the active layers.

increases linearly with incident energy and with the fineness of the sampling. Thus:

$n_{ch} \propto E/t$ (*t* is the thickness of each absorber layer)

For statistically independent sampling the sampling contribution to the stochastic term is:

$$\sigma_{samp}/E \propto 1/\sqrt{n_{ch}} \propto \sqrt{(t/E)}$$

Thus the resolution improves as *t* is decreased.

For EM order 100 samplings required to approach the resolution of typical homogeneous devices ➔ impractical.

Typically: $\sigma_{samp}/E \sim 10\%/\sqrt{E}$

EM calorimeters: energy resolution

Homogeneous calorimeters: all the energy is deposited in an active medium.
Absorber ≡ active medium → All e+e- over threshold produce a signal
Excellent energy resolution

Compare processes with different energy threshold

Scintillating crystals

$$E_s \cong \beta E_{gap} \sim eV$$

$$\approx 10^2 \div 10^4 \gamma / MeV$$

$$\sigma / E \sim (1 \div 3)\% / \sqrt{E(GeV)}$$

Lowest possible limit

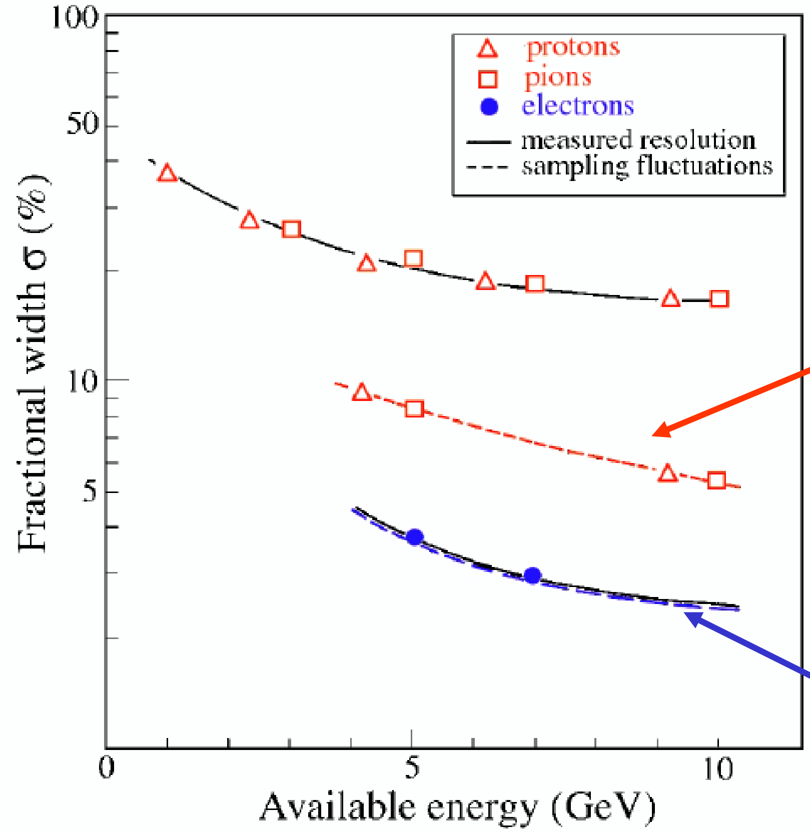
Cherenkov radiators

$$\beta > \frac{1}{n} \rightarrow E_s \sim 0.7 MeV$$

$$\approx 10 \div 30 \ \gamma / MeV$$

$$\sigma / E \sim (10 \div 5)\% / \sqrt{E(GeV)}$$

Sampling fluctuations in EM and hadronic showers



sampling fluctuations only minor contribution to hadronic resolution in non-compensating calorimeter

EM resolution dominated by sampling fluctuations

FIG. 4.15. The energy resolution and the contribution from sampling fluctuations to this resolution measured for electrons and hadrons, in a calorimeter consisting of 1.5 mm thick iron plates separated by 2 mm gaps filled with liquid argon. From [Fab 77].

Fluctuations in hadronic showers

Some types of fluctuations as in EM showers, plus:

1) Fluctuations in visible energy
   (ultimate limit of hadronic energy resolution)

2) Fluctuations in the EM shower fraction, f_{em}
   - Dominating effect in most hadron calorimeters ($e/h \neq 1$)
   - Fluctuations are asymmetric in pion showers (one-way street)
   - Differences between p, π induced showers
     No leading π^0 in proton showers (barion # conservation)

1) Fluctuations in visible energy

Fluctuations in losses due to nuclear binding energy

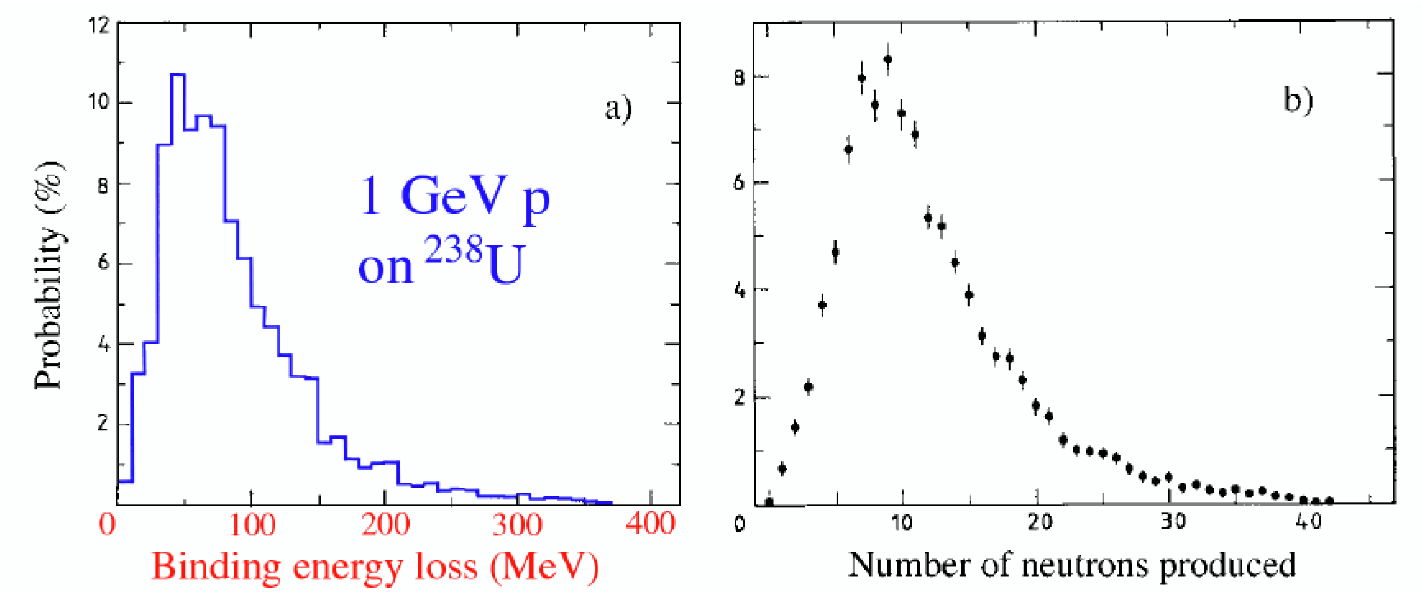


FIG. 4.43. The nuclear binding energy lost in spallation reactions induced by 1 GeV protons on ^{238}U nuclei (*a*), and the number of neutrons produced in such reactions (*b*). From [Wig 87].

- Estimate of the fluctuations of nuclear binding energy loss in high-Z materials ~15%
- Note the strong correlation between the distribution of the binding energy loss and the distribution of the number of neutrons produced in the spallation reactions
- There may be also a strong correlation between the kinetic energy carried by these neutrons and the nuclear binding energy loss

2) Fluctuations in the EM shower fraction

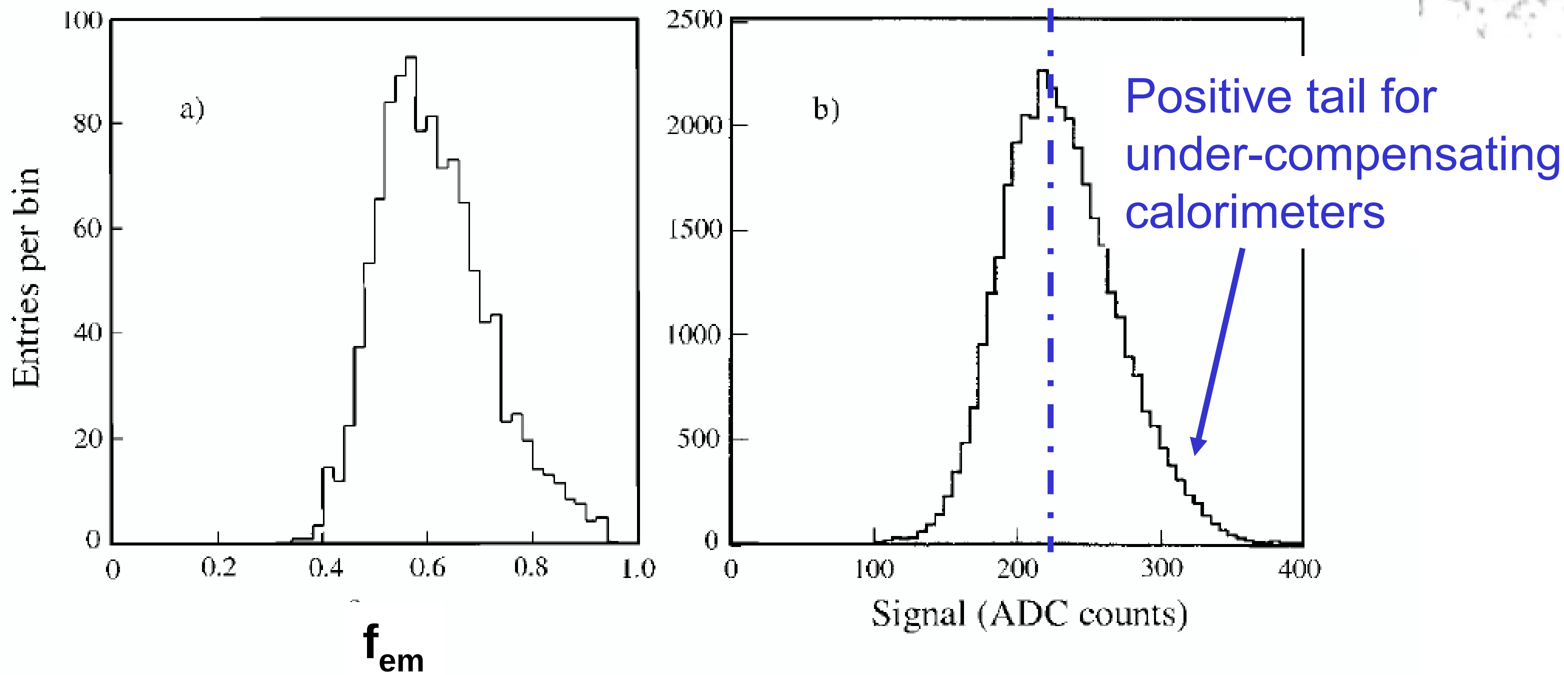


FIG. 4.44. The distribution of the fraction of the energy of 150 GeV π^- showers contained in the em shower core, as measured with the SPACAL detector (a) [Aco 92b] and the signal distribution for 300 GeV π^- showers in the CMS Quartz-Fiber calorimeter (b) [Akc 98].

Pion showers: Due to the irreversibility of the production of π_0s and because of the leading particle effect, there is an asymmetry in the probability that an anomalously large fraction of the energy goes into the EM shower component

Differences in p / π induced showers

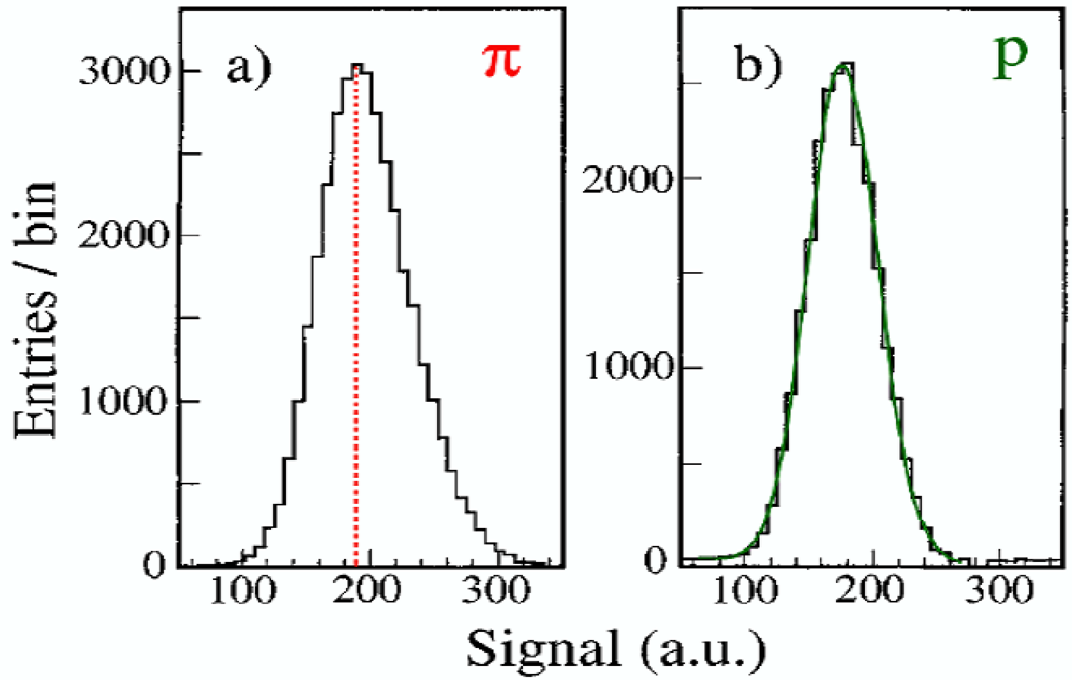


FIG. 4.49. Signal distributions for 300 GeV pions (*a*) and protons (*b*) detected with a quartz-fiber calorimeter. The curve represents the result of a Gaussian fit to the proton distribution [Akc 98].

<f_{em}> is smaller in proton-inducer showers than in pion induced ones: barion number conservation prohibits the production of leading π_0s and thus reduces the EM component respect to pion-induced showers

Measure f_{em}

Ideal:

measure f_{em} for each event and weight EM and hadronic part of shower differently

- ➔ dual readout: separate measurement of EM fraction using quartz in addition to scintillators as active media
- ➔ very high granularity + software decomposition of shower with appropriate clustering algorithm

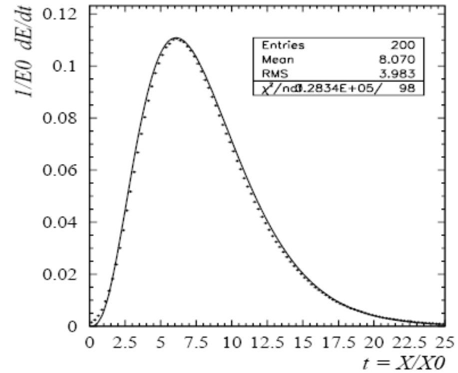
Practically:

- ➔ for many calorimeters neither solution is viable
- ➔ try energy density weighting techniques

Shape analysis: longitudinal

The parameterization of EM shower longitudinal development with gamma distribution function was proposed in 1975[1].

$$\frac{dE}{dx} = E \cdot \frac{1}{\lambda \cdot \Gamma(\alpha)} \cdot \left(\frac{x}{\lambda}\right)^{\alpha-1} \cdot e^{-x/\lambda}$$



Gamma function:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$$

Later the similar parameterization was introduced for hadronic showers[2] as the following 2-component function:

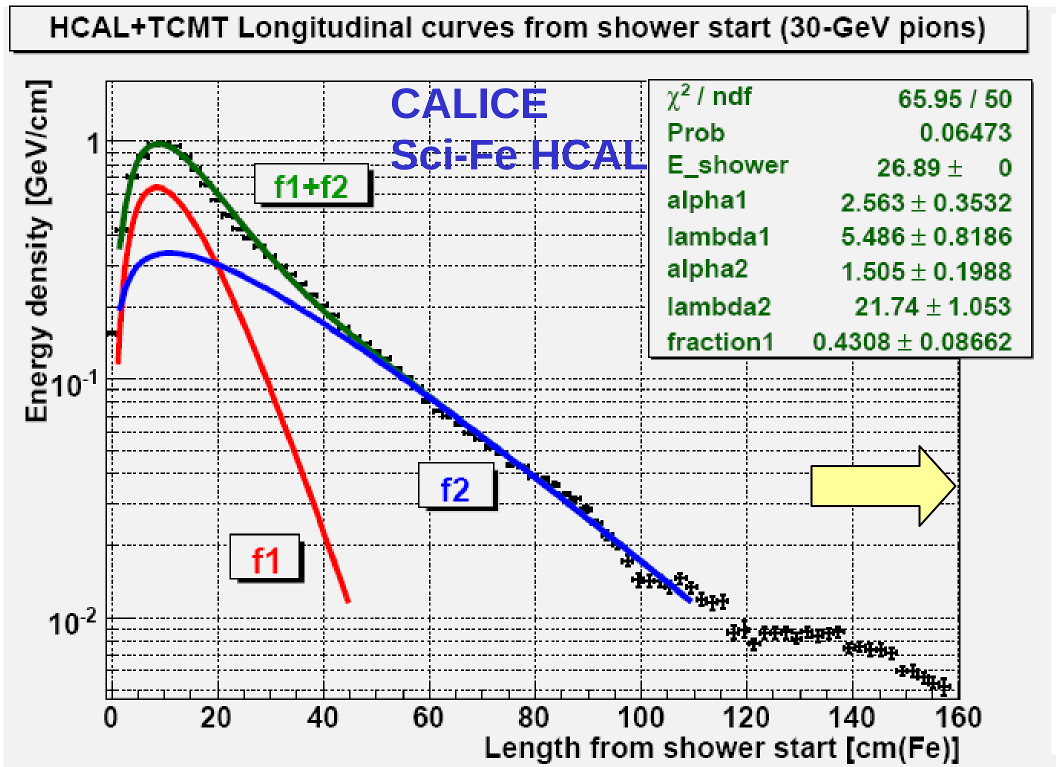
$$\frac{dE}{dx} = E \cdot \left(\frac{w}{\lambda_1 \cdot \Gamma(\alpha_1)} \cdot \left(\frac{x}{\lambda_1}\right)^{\alpha_1 - 1} \cdot e^{-x/\lambda_1} + \frac{1-w}{\lambda_2 \cdot \Gamma(\alpha_2)} \cdot \left(\frac{x}{\lambda_2}\right)^{\alpha_2 - 1} \cdot e^{-x/\lambda_2} \right) = f_1 + f_2$$

where *w* is the EM and *1-w* the hadronic fraction of hadronic shower

[1] E.Longo and I. Sestili, NIM, 128 (1975), 283.
[2] R.K. Bock et al. NIM, 186 (1981), 533.

f_{em} from longitudinal shower profile

$$\frac{dE}{dx} = E\cdot\left(\frac{w}{\lambda_1\cdot\Gamma(\alpha_1)}\cdot\left(\frac{x}{\lambda_1}\right)^{\alpha_1-1}\cdot e^{-x/\lambda_1} + \frac{1-w}{\lambda_2\cdot\Gamma(\alpha_2)}\cdot\left(\frac{x}{\lambda_2}\right)^{\alpha_2-1}\cdot e^{-x/\lambda_2}\right) = f_1 + f_2$$



$w \propto f_{em}$

Beam energy GeV	Shower energy GeV	w	λ_1 cm	λ_2 cm
Protons				
30	25.4	0.31	5.4	21.9
40	34.4	0.22	4.2	21.3
50	43.6	0.30	4.7	20.2
80	69.8	0.49	6.2	17.2
π^+				
30	27.0	0.46	5.7	22.1
40	36.1	0.47	5.9	17.9
50	45.5	0.55	5.9	17.8
80	71.7	0.62	6.2	15.6

- f_{em} increases with increasing energy of shower particle
- larger f_{em} for pion than for protons

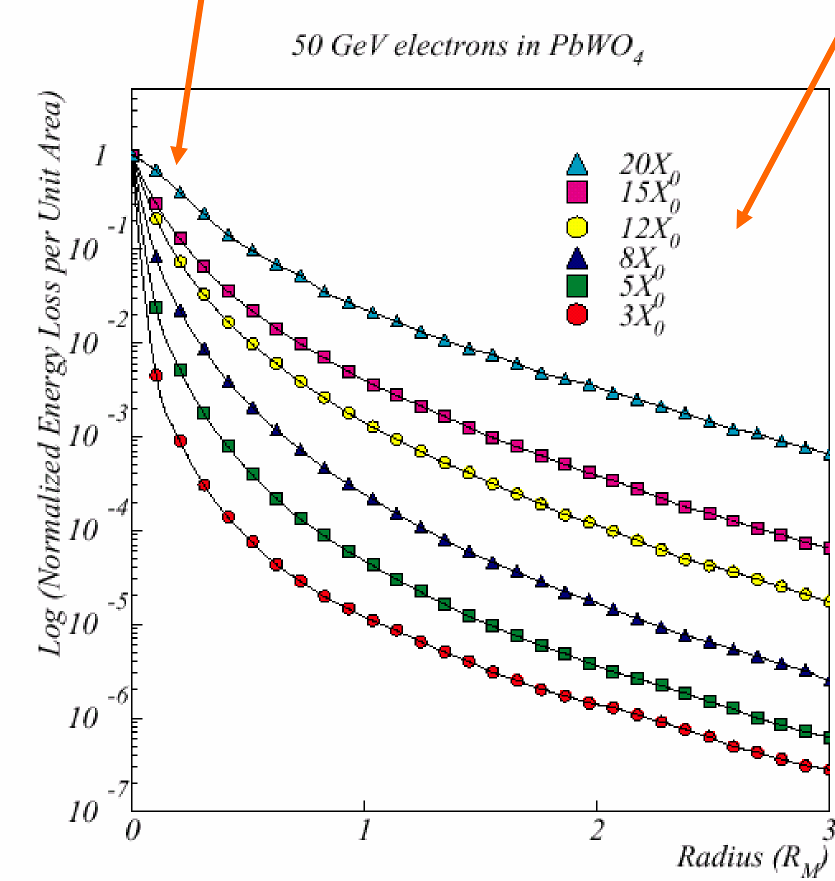
Large fluctuations in fem event by event are not reflected in this mean numbers!!!

Lateral profile of EM showers

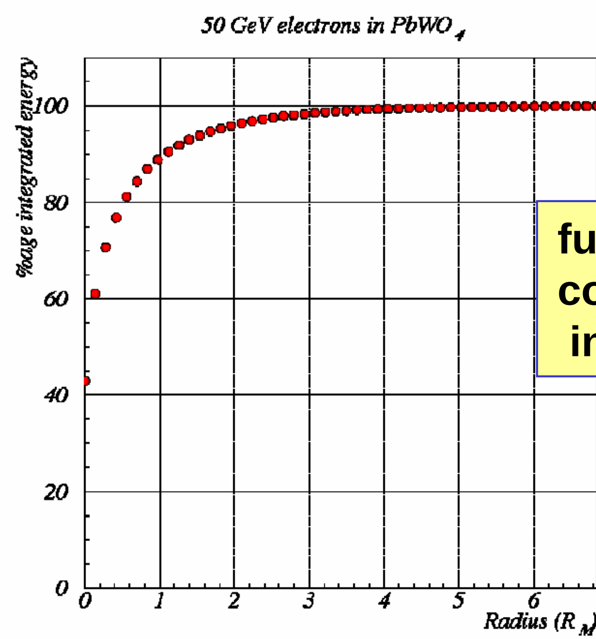
Generally 2 fit components:

Central core: multiple scattering

Peripheral halo:
propagation of less attenuated photons, widens with depth the shower



$$\frac{dE}{dr} = N \cdot \left(p\left(\frac{1}{\lambda_1}\right) \cdot e^{-r/\lambda_1} + (1-p)\left(\frac{1}{\lambda_2}\right) \cdot e^{-r/\lambda_2} \right)$$



full EM containment in 5 R_M

f_{em} from lateral shower shape

Use same parameterization as for EM shower to identify core component ($\propto f_{em}$) and peripheral component of hadronic shower

$$\frac{dE}{dr} = N \cdot \left(p \left(\frac{1}{\lambda_1} \right) \cdot e^{-r/\lambda_1} + (1-p) \left(\frac{1}{\lambda_2} \right) \cdot e^{-r/\lambda_2} \right)$$

$\lambda_1 = 2$ cm

$\lambda_2 = 8$ cm

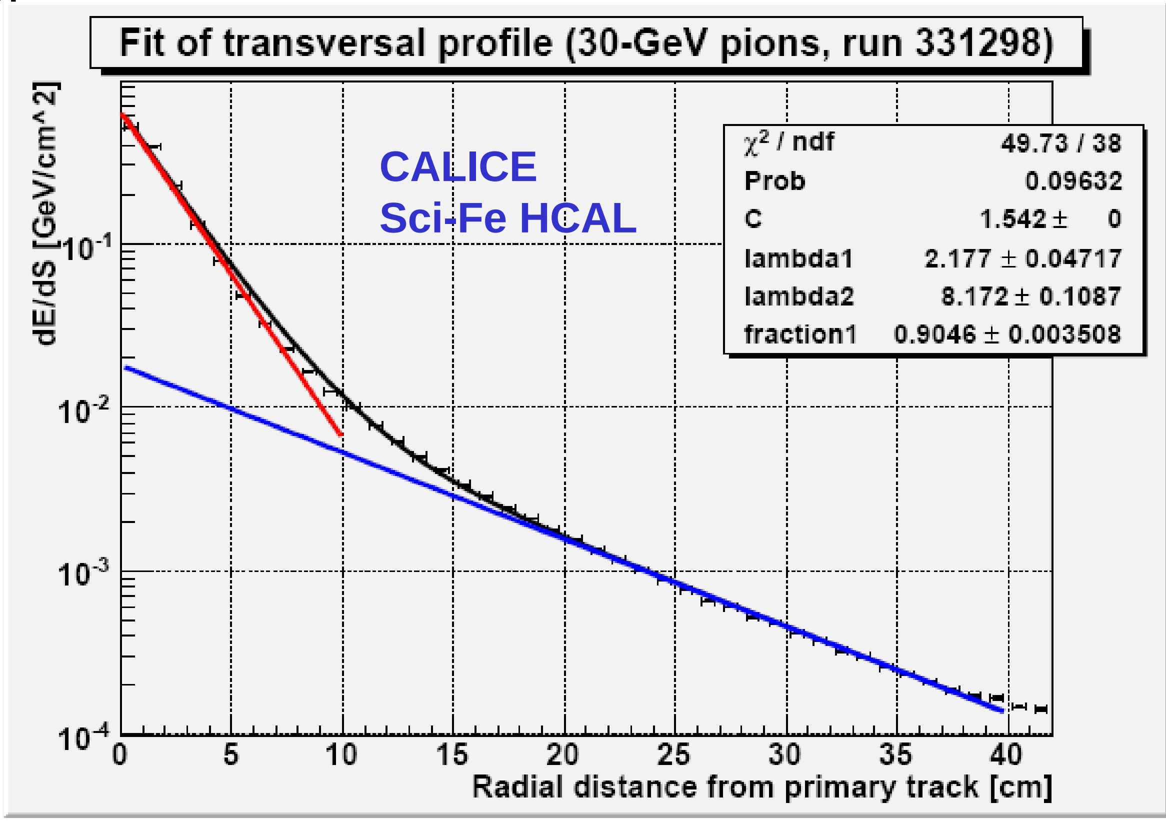
Fe:

$R_M = 1.8$ cm

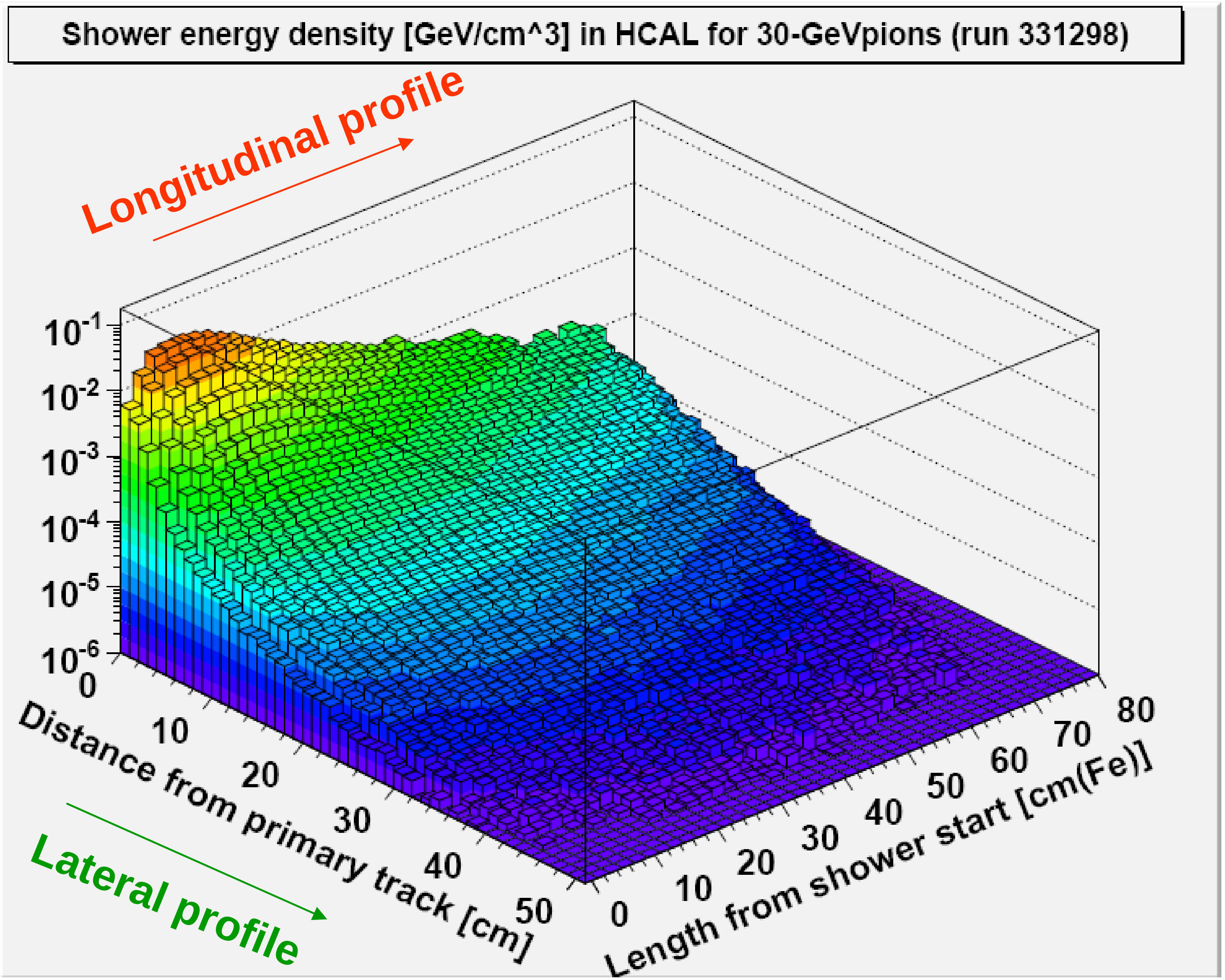
$\lambda_{int} = 16$ cm

Important:

Lateral and longitudinal profiles are strongly coupled i.e. wider profile at shower max



Shower shape: lateral & longitudinal



Summary: Fluctuations in hadronic showers

Some types of fluctuations as in EM showers, plus:

1) Fluctuations in visible energy
   (ultimate limit of hadronic energy resolution)

2) Fluctuations in the EM shower fraction, f_{em}

Energy resolution of hadron showers

Hadronic energy resolution of non-compensating calorimeters does not scale with $1/\sqrt{E}$

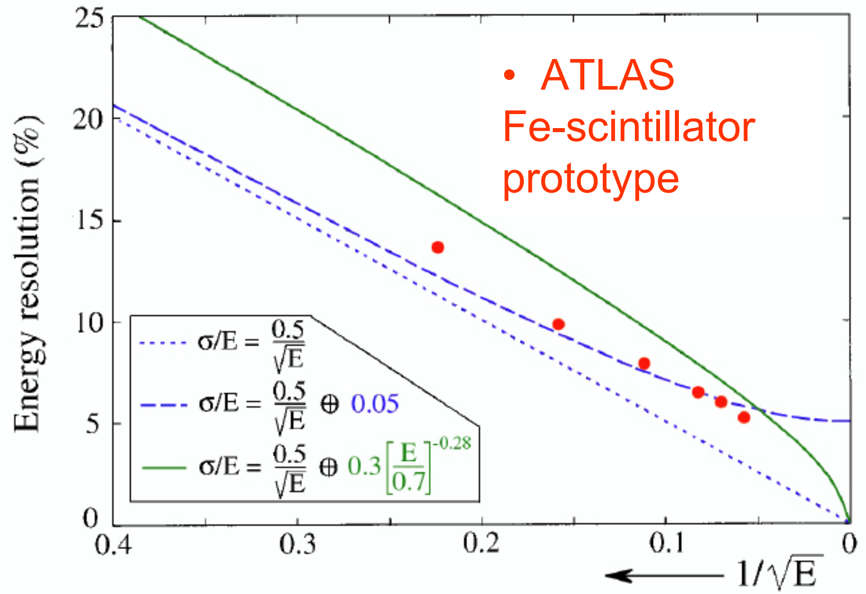
➔ $\sigma / E = a/\sqrt{E} \oplus b$ does not describe the data

Effects of non-compensation on σ/E is are better described by an energy dependent term:

$$\sigma / E = a/\sqrt{E} \oplus b\,(E/E_0)^{L-1}$$

In practice a good approximation is:

$$\sigma / E = a/\sqrt{E} + b$$



E resolution winners: crystal calorimeters

Among different types of calorimeters those with scintillating crystals are the most precise in energy measurements

- Excellent energy resolution (over a wide range)
- High detection efficiency for low energy e and γ
- Structural compactness:
  - simple building blocks allowing easy mechanical assembly
  - hermetic coverage
  - fine transverse granularity
- Tower structure facilitates event reconstruction
  - straightforward cluster algorithms for energy and position
  - electron/photon identification
- Perfect for EM calorimeters, impossible to use for high energy hadron calorimeters

Compensating calorimeters

Sampling fluctuations also degrade the energy resolution.

As for EM calorimeters: $\sigma_{samp}/E \propto \sqrt{t}$ where t is the absorber thickness

(empirically, the resolution does not improve for t ≲ 2 cm (Cu))

ZEUS at HERA employed an intrinsically compensated ^{238}U/scintillator calorimeter

The ratio of ^{238}U thickness (3.3 mm) to scintillator thickness (2.6 mm) was tuned such that $e/\pi = 1.00 \pm 0.03$

For this calorimeter:

$$\sigma_{intr}/E = 26\%/\sqrt{E} \qquad \text{and} \qquad \sigma_{samp}/E = 23\%/\sqrt{E}$$

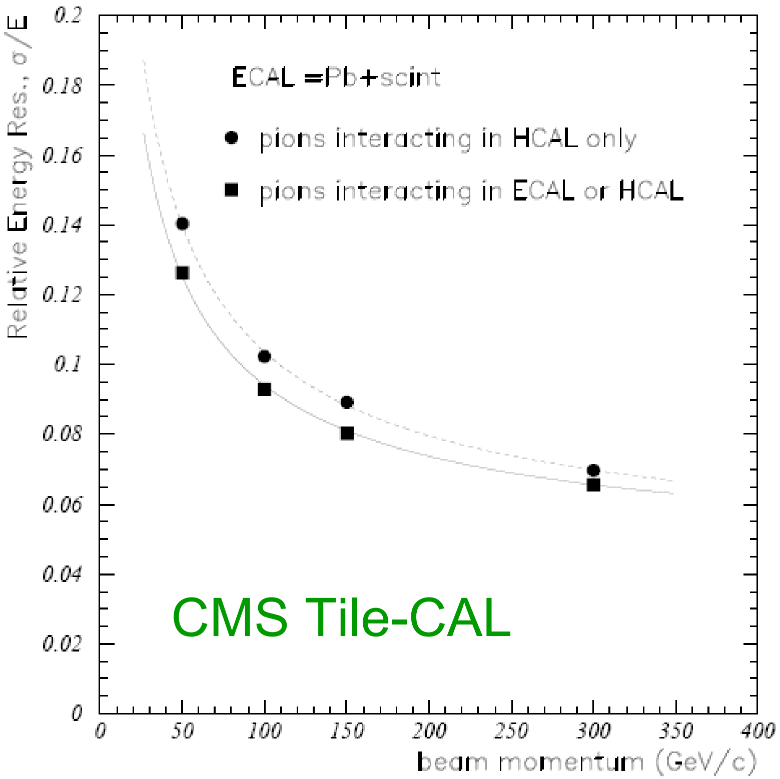
Giving an excellent energy resolution for hadrons:

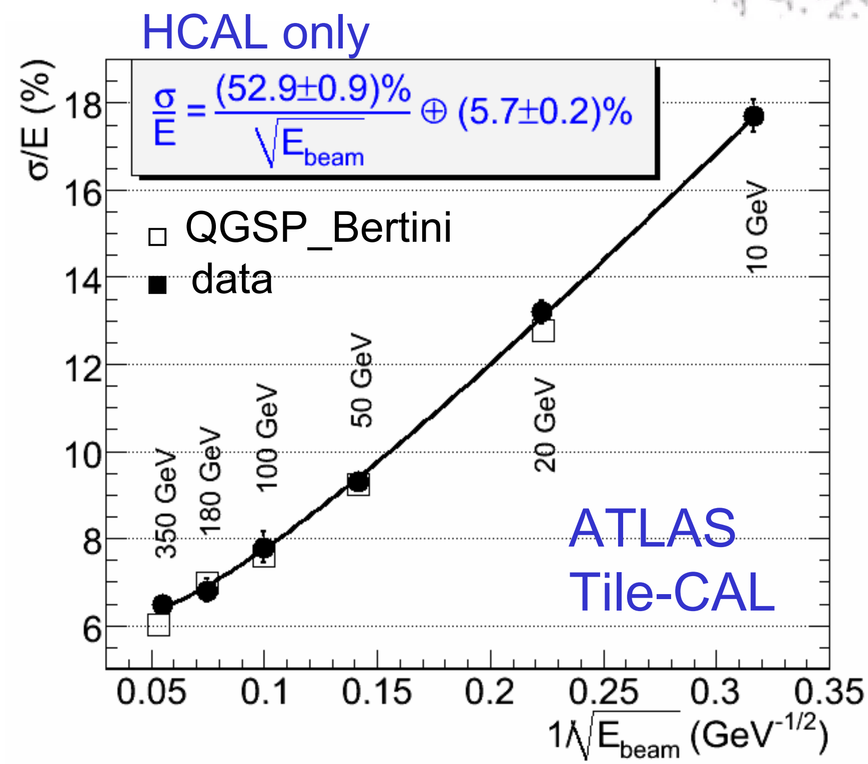
$$\sigma_{had}/E \sim 35\%/\sqrt{E}$$

The downside is that the ^{238}U thickness required for compensation (~ $1X_0$) led to a rather modest EM energy resolution:

$$\sigma_{EM}/E \sim 18\%/\sqrt{E}$$

Examples: HCAL E resolution





HCAL only

$\sigma/E = (93.8 \pm 0.9)\%/\sqrt{E} \oplus (4.4 \pm 0.1)\%$

ECAL+HCAL

$\sigma/E = (82.6 \pm 0.6)\%/\sqrt{E} \oplus (4.5 \pm 0.1)\%$

Improved resolution using full calorimetric system (ECAL+HCAL)

ATLAS LAr + Tile for pions: $\frac{\sigma(E)}{E} = \frac{42\,\%}{\sqrt{E}} \oplus 2\,\%$

What is really needed in terms of E res.?

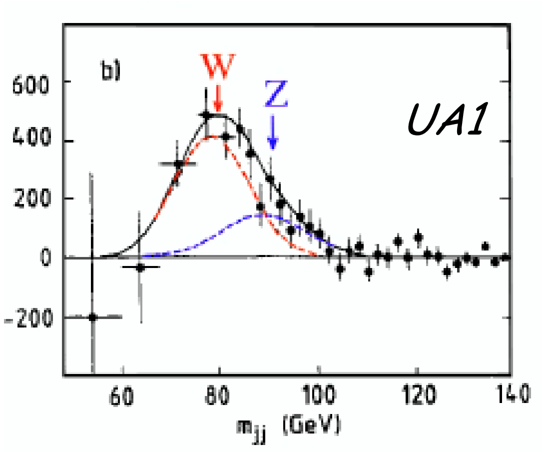
1) Hadron energy resolution can be improved with weighting algorithms
   - what is the limit?

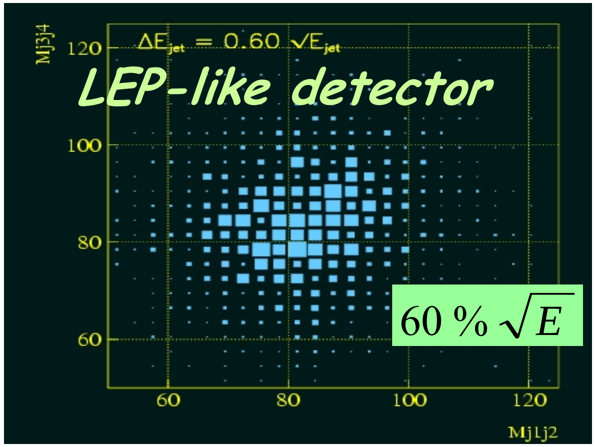
2) HEP experiments measure jets, not single hadrons (?)
   - How does the jet energy resolution relate to the hadron res.?

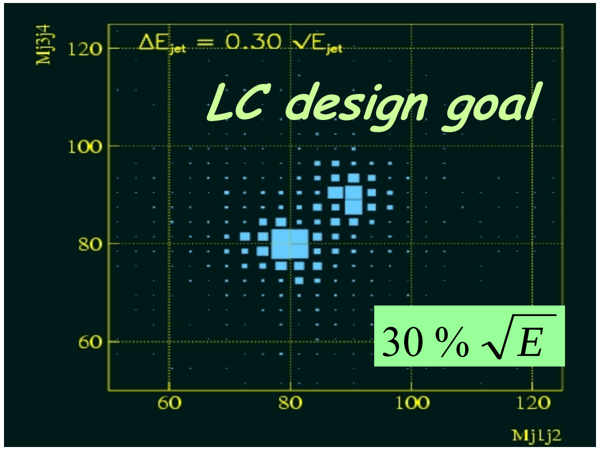
3) Jet energy resolution depends on whole detector and only partially on HCAL performance ➔ Particle Flow
   - What is the true hadron energy resolution required?

4) What is the ultimate jet energy resolution achievable?
   - Dual readout better than PFlow?

Challenge: W Z separation



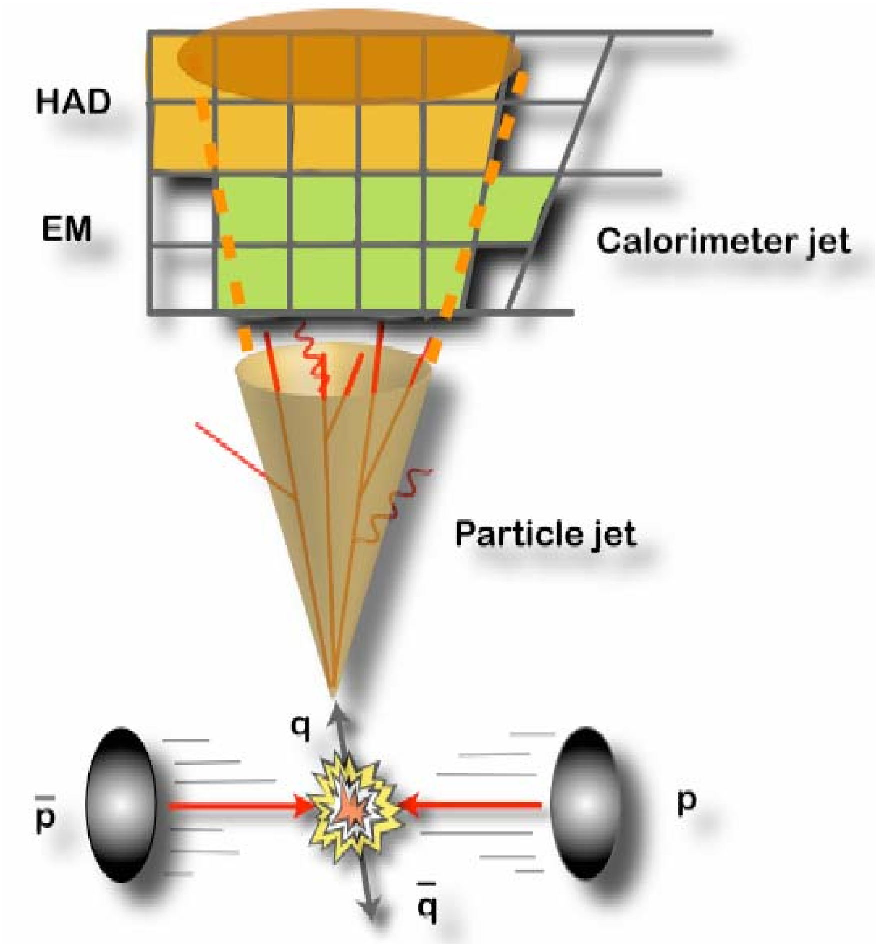




At the Tera-scale, we need to do physics with W's and Z's as Belle and Babar do with D^+ and D_s

Calorimeter performance for jets has to improve by a factor 2 w.r.t. LEP

From single hadrons to jets



- **Calorimeter jet (cone)**
  - jet is a collection of energy deposits with a given cone *R*: $R = \sqrt{\Delta\varphi^2 + \Delta\eta^2}$
  - cone direction maximizes the total E_T of the jet
  - various clustering algorithms

Time

→ correct for finite energy resolution
→ subtract underlying event
→ add out of cone energy

- **Particle jet**
  - a spread of particles running roughly in the same direction as the parton after hadronization

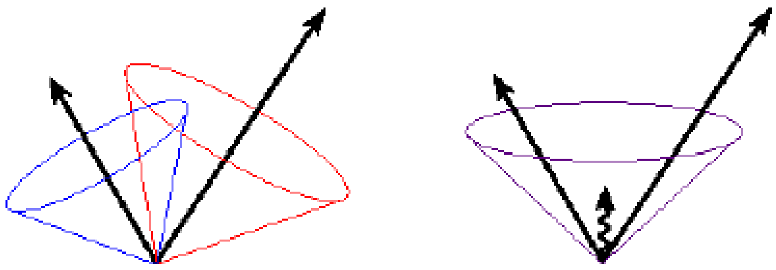
Jet = sum of many particles (e,γ,π,p,n,K,…) produced in the fragmentation of a hadron.

technically: charged particles in tracker + ECAL + HCAL clusters + E_{miss}

Example of Jet algorithm: cone

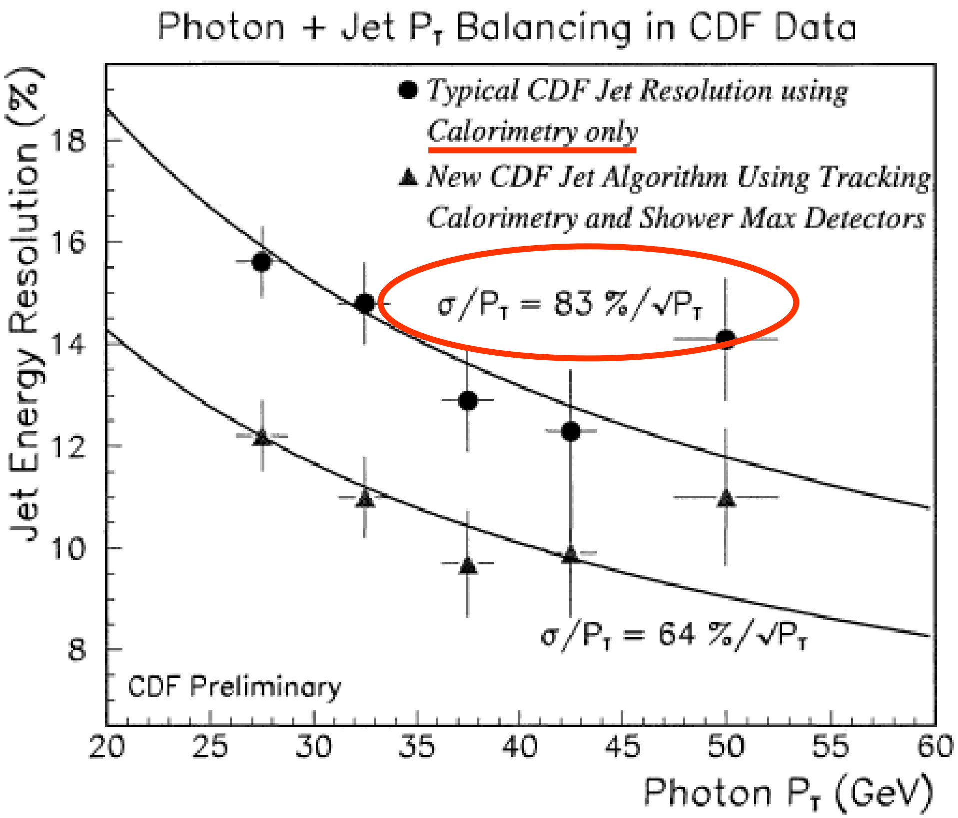
- Cone algorithms
  - draw a cone of fixed size around a seed
  - compute jet axis
  - draw a new cone around the new jet axis and recalculate axis and new E_T
  - iterate until stable

  ➜ In addition:
  - add additional midpoint seeds between pairs of close jets
  - split/merge after stable proto-jets found



Jets at CDF

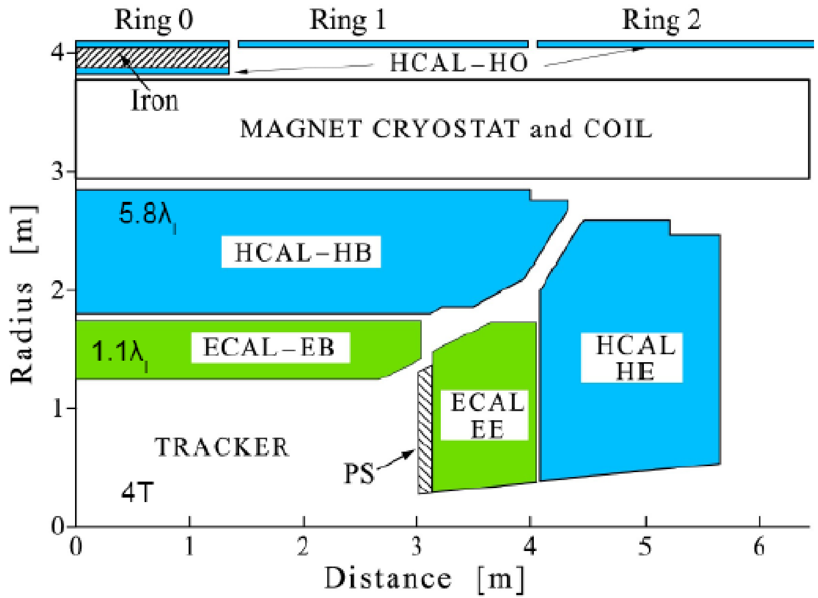
		Central	Plug
EM	thickness	$19\,X_0$, 1λ	$21\,X_0$, 1λ
	sample(Pb)	$0.6\,X_0$	$0.8\,X_0$
	sample(scint.)	5 mm	4.5 mm
	wavelength sh.	sheet	fiber
	resolution	$\frac{13.5\%}{\sqrt{E_T}} \oplus 2\%$	$\frac{14.5\%}{\sqrt{E}} \oplus 1\%$
HAD	thickness	$4.5\,\lambda$	$7\,\lambda$
	sample(Fe)	25-50 mm	50 mm
	sample(scint.)	10 mm	6 mm
	wavelength sh.	finger	fiber
	resolution	$\frac{50\%}{\sqrt{E_T}} \oplus 3\%$	$\frac{70\%}{\sqrt{E}} \oplus 4\%$



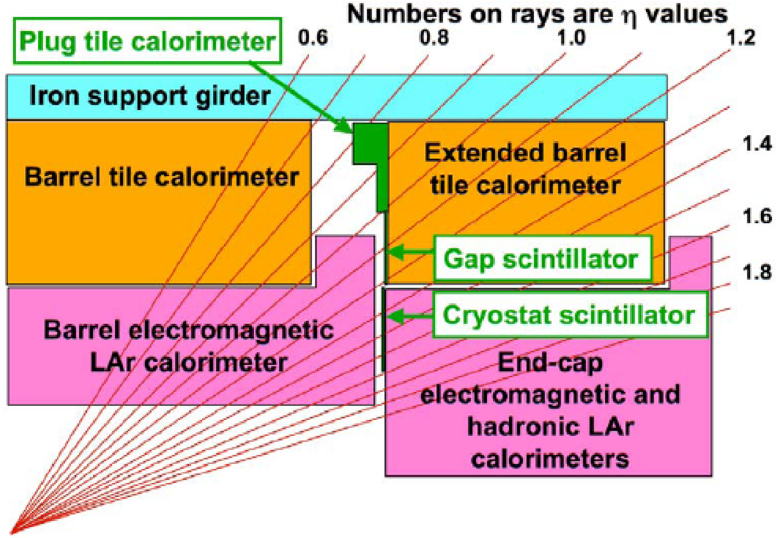
Jet energy performance in calorimeter worse than hadron performance

Examples: jet energy resoltuion

CMS



ATLAS



5 cm brass / 3.7 cm scint.
Embedded fibres, HPD readout

Expected jet resolution:

$$\frac{\sigma}{E} = \frac{125\%}{\sqrt{E}} \oplus \frac{5.6\ \mathrm{GeV}}{E} \oplus 3.3\%$$

14 mm iron / 3 mm scint.
sci. fibres, read out by phototubes

Jet resolution with weighting:

$$\frac{\sigma}{E} = \frac{60\%}{\sqrt{E}} \oplus 3\%$$

Stochastic term for hadrons was ~93% and 42% respectively

Effect of Jet algorithm (ATLAS)

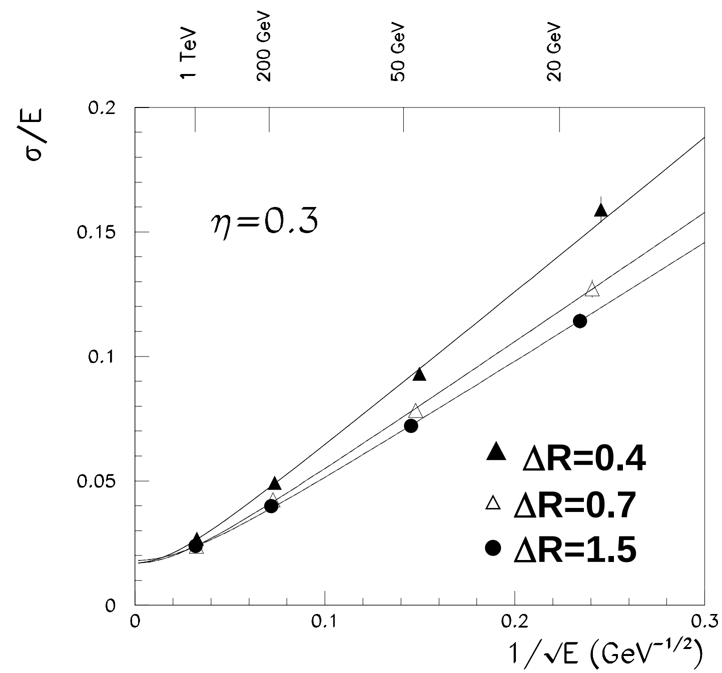
Cone Algorithm

- Highest E_T tower for jet seed + cone
- Iteration of cone direction, jet overlap, energy sharing, merging

Cone size influence on reconstructed jet energy and resolution

$$\sigma / E = a / \sqrt{E} \oplus b$$

	a (%GeV$^{1/2}$)	b (%)
Full Calo	48.2 ± 0.9	1.8 ± 0.1
ΔR=0.7	52.3 ± 1.1	1.7 ± 1.1
ΔR=0.4	62.4 ± 1.4	1.7 ± 0.2



Summary on calorimeter response

To make a statement about the energy of a particle:

1. relationship between measured signal and deposited energy (response = average signal per unit of deposited energy)

2. energy resolution (precision with which the unknown energy can be measured)
   - ➔ dominated by fluctuations especially for the hadronic case
   - ➔ Jet E res. normally worse than E res. of single hadrons
   - ➔ can generally be improved by software weighting techniques

Next: - software compensation = the role of high granularity
- energy weighting

Improving the calorimeter response
weighting techniques

How to improve the calorimeter response ➔ fight against fluctuations !!!

Two main issue go under the same name of "weighting":

1. Correction for layers with different sampling fractions

   maintain response linearity when adding energy from different sub-detectors or calorimeter blocks.

➔ relevant for EM and hadronic showers

2. Software compensation

   improve energy resolution of hadronic shower by correcting the pure hadronic component for e/h differences and for invisible energy losses

➔ relevant for hadronic showers / jets

Sampling calorimeters: layer weights

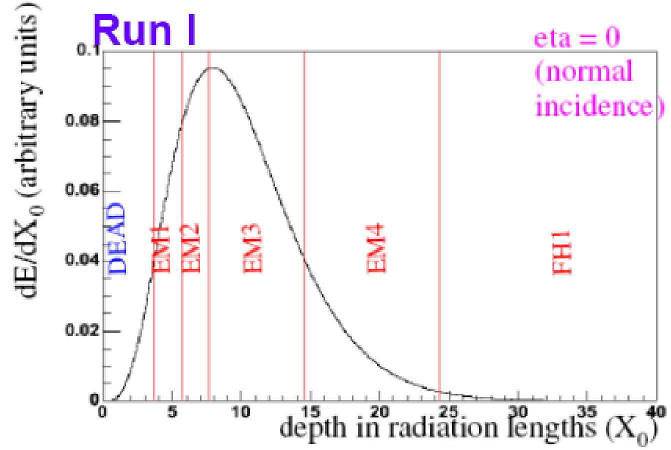
Issue: preserve response linearity in the calorimeter system

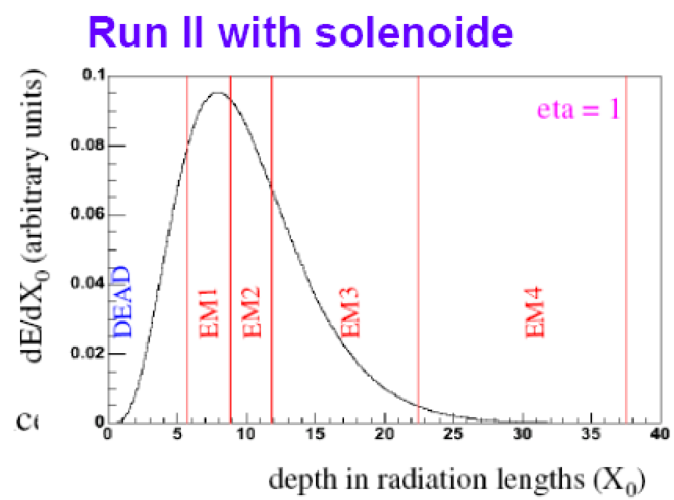
For sampling calorimeters, the signal deposited in each active layer has to be multiplied by an adequate factor to get back the “true” energy

Factors are determined by test beam and/or simulation

The weights for the early layers have to take into account the losses due to dead material in front of the calorimeter

According to the angle of the incident particle, the amount of dead material varies layer weights vary

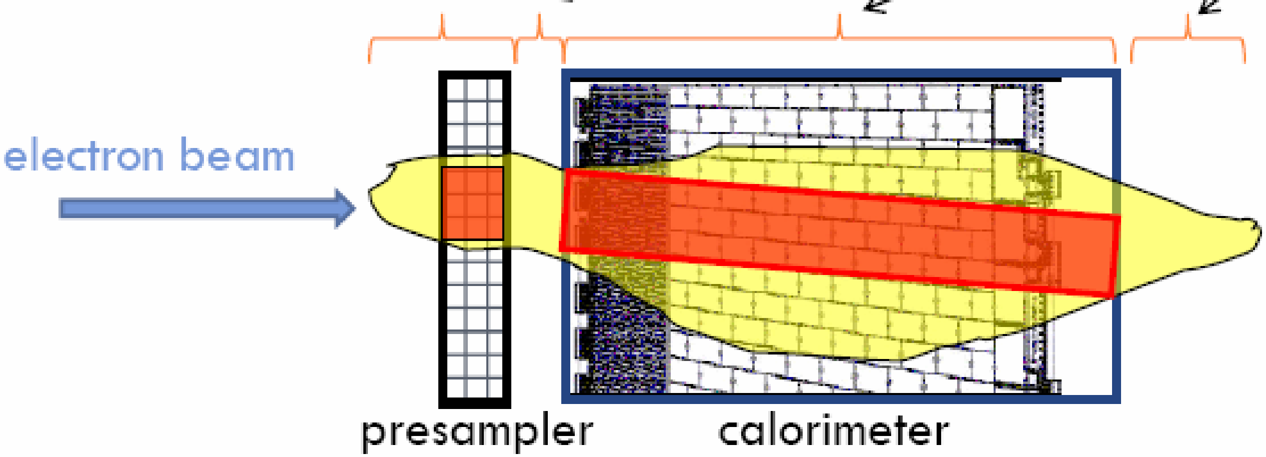


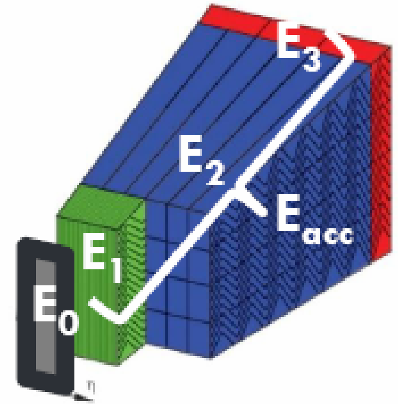


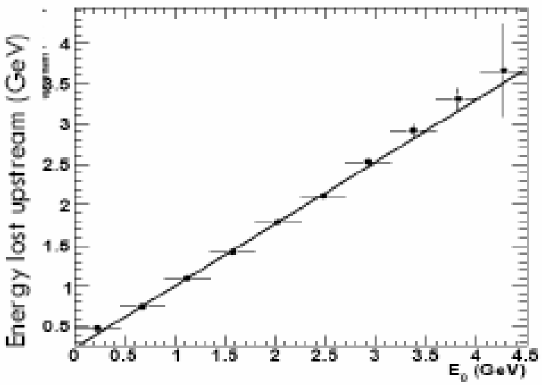
Electron energy calibration

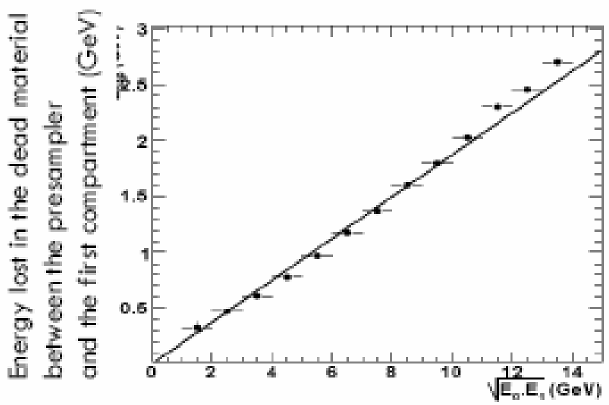
- Energy parameterisation:

$$E_{electron} = \text{offset} + W_0 E_0 + W_{01}\sqrt{E_0 E_1} + \lambda E_{acc} + W_3 E_3$$







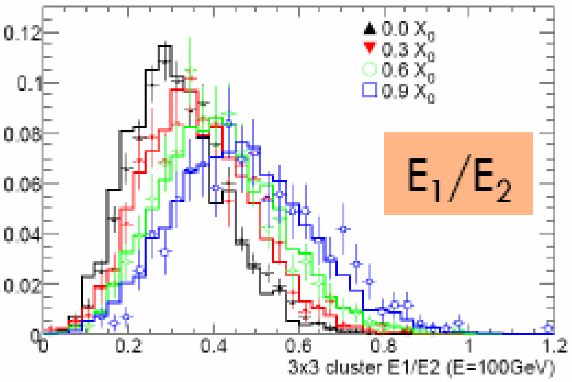


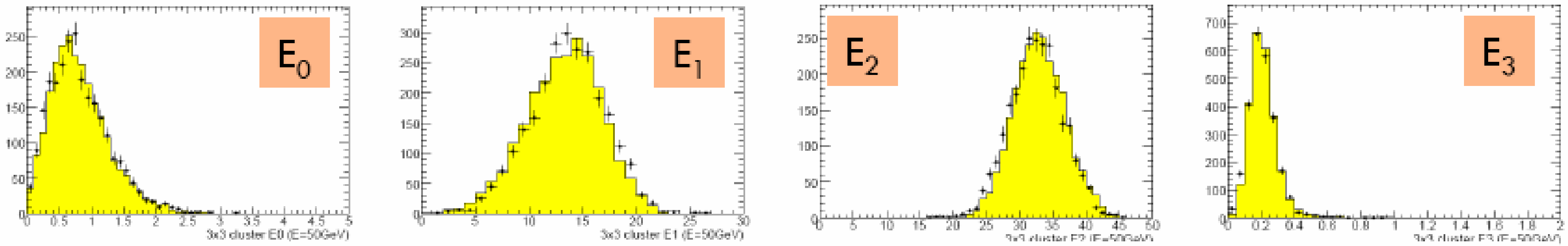
- Offset: energy lost by ionisation in the dead material in front of the calorimeter.
- W_0: correcting for energy lost in front of calorimeter by pre-showering electrons.
- W_{01}: empirical correction for the energy lost

  in the dead material between the presampler and the first compartment.
- λ: out of cluster correction and sampling fraction
- W_3: correcting for the energy leakage at the back of the calorimeter

Data/MC comparisons

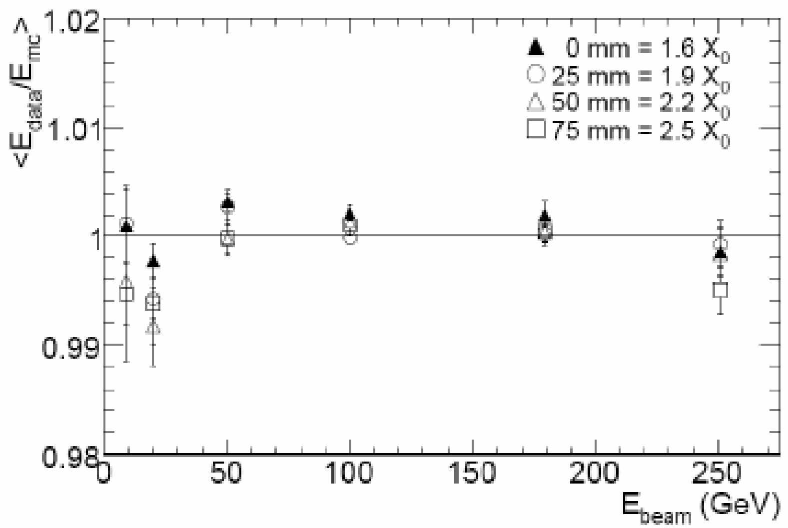
- The energy calibration strategy of the LAr calorimeter relies on the simulation of the experimental set-up and the exact description of the detector response
- A high level of agreement between data and MC is therefore crucial for the performance of the detector.
- In the CTB a big emphasis was given to a careful data-MC comparison.





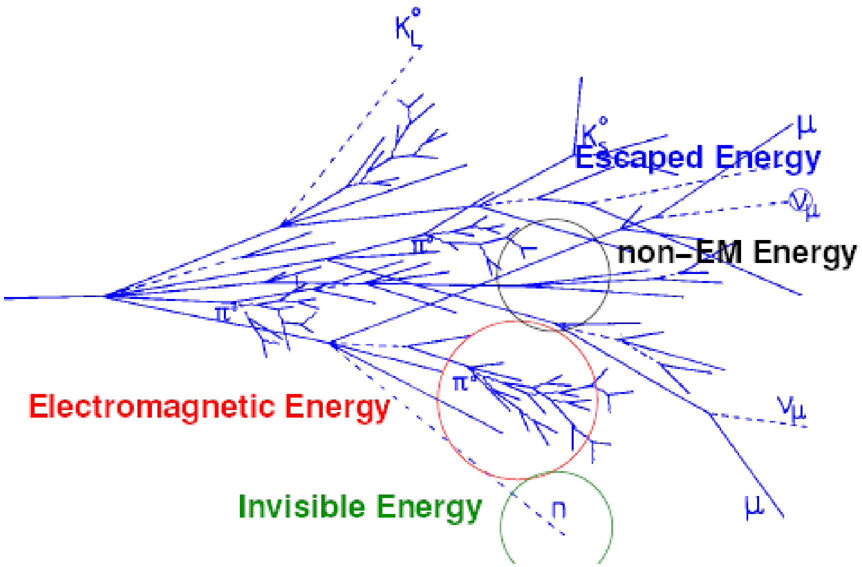
- Percentage mean energy difference between data and MC simulation for all energies and all material configurations

  Considering all systematic errors, the level of agreement between the MC and the data was estimated to be of order 0.4%



Hadron shower components

A hadronic shower consists of:



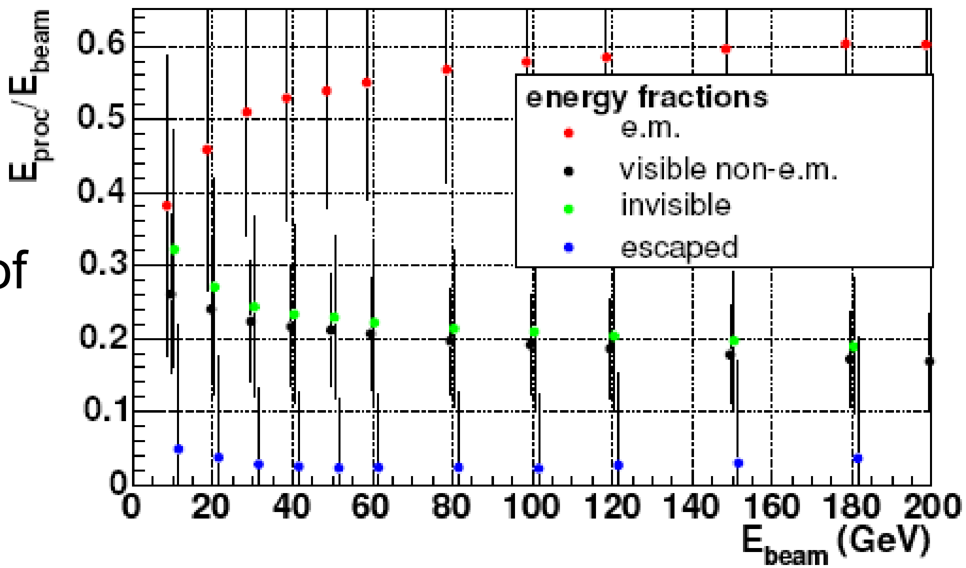
$$E_p = f_{em} e + (1 - f_{em}) h$$

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$

• each fraction is energy dependent and subject to large fluctuations

• invisible energy is the main source of the non-compensating nature of hadron calorimeters

• hadronic calibration has to account for the invisible and escaped energy



Energy density weighting

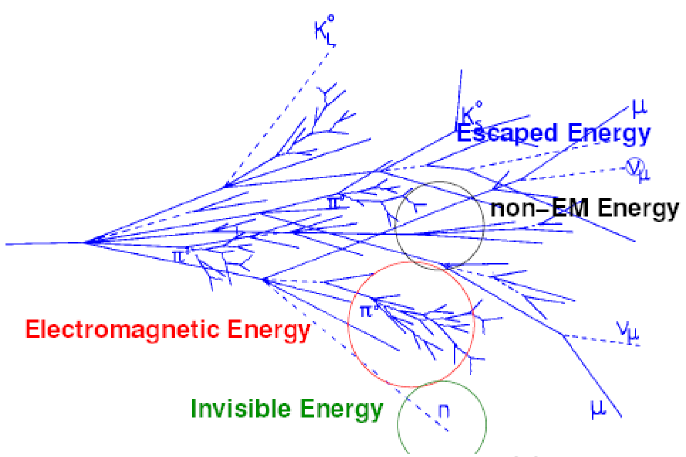
IDEA:

•separate EM part of the shower from the non-EM part

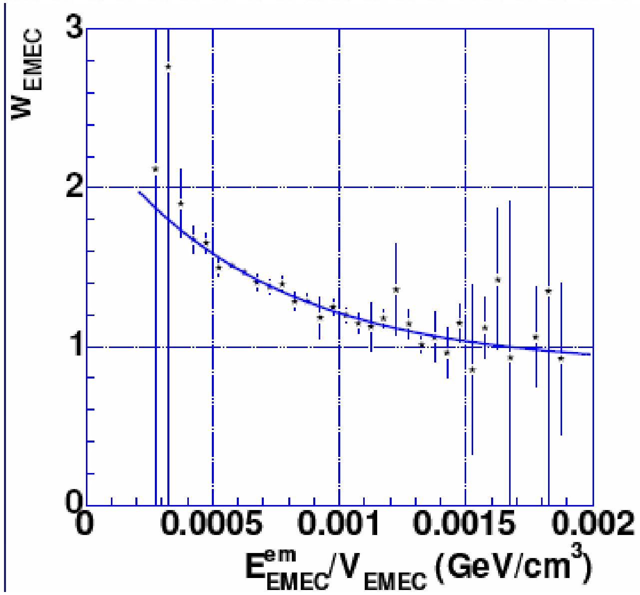
•apply a weight to the non-EM part to compensate different response (e/h) and invisible energy

How to separate EM fraction from non-EM fraction?

- X_0 O(1-2cm) << λ O(20cm)
- high energy density (energy in a cell) denotes high EM activity
- low energy density corresponds to hadronic activity
- apply weights as function of energy density



H1 weighting method



$E' = w\,E$

$w = [c_1 \exp(-c_2\, E/V) + c_3]$

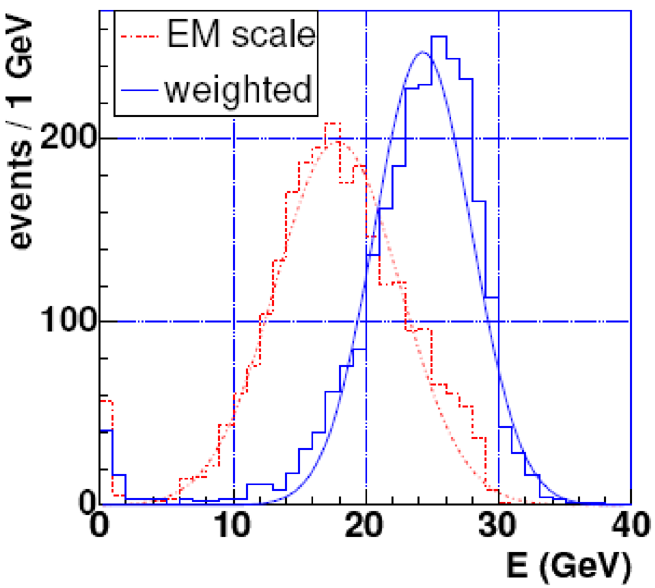
different definitions of the volume possible

w → 1 for large *E/V (EM case)*:

- $c_3 \sim 1$
- weighting does not change electromagnetic clusters

small energy density dominated by hadronic activity: $w > 1$:

- $c_{1,2} > 0$
- exact values depend on total cluster energy, choice of weighted unit (cell or cluster), . . .



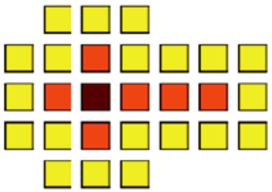
← 30 GeV pions from ATLAS test beam as a simple cluster weight example

- improved E scale and resolution after weighting

H1 weighting method @ clusters level

$$E'_{\text{sub-calo}} = w\, E_{\text{sub-calo}}$$
$$w = [c_1 \exp(-c_2\, E_{\text{sub-calo}}/V_{\text{sub-calo}}) + c_3]$$

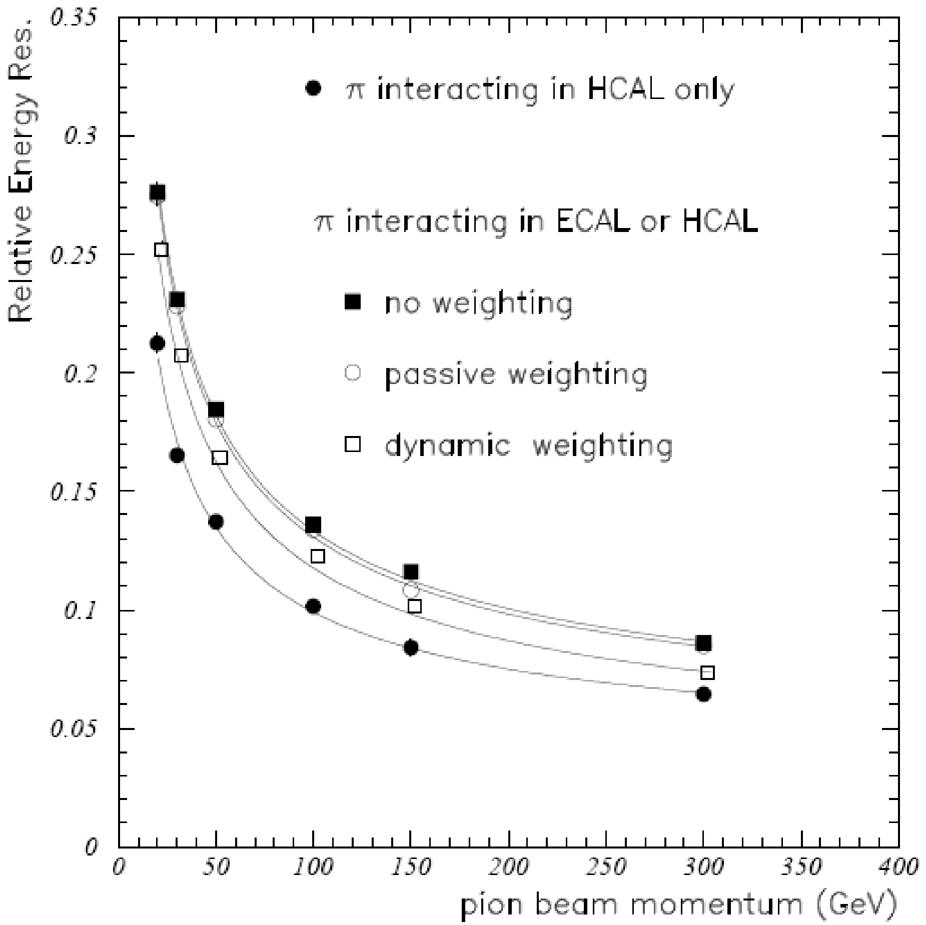
- reconstruct “3D”-cluster



Cluster:
- a group of calorimeter cells topologically connected
- often grouped around a seed cell with some large energy
- either fixed in size or dynamic
- should be the base for hadronic calibration

- split the cluster in sub-calorimeter parts (ECAL 1, ECAL 2, HCAL)
  because weights depend on intrinsic calorimeter properties
- apply cluster-energy dependent weights found in test beam as function of $E_{\text{sub-calo}}/V_{\text{sub-calo}}$
- tested on single particle test beam data and MC only
  no straightforward extension to jets :-(
  serves as a simple test case for H1 weighting
  does not need any MC as input :-)

Energy weighting @ cluster level (CMS)



passive weighting (sampling):
increase the weight of the 1st HCAL readout segment by an energy independent constant

$$\sigma(E)/E = 122\%/E^{1/2} \oplus 5\%$$

dynamic weighting (energy w.):
event-by-event correction dependent on the fraction of the energy deposited in the 1st readout segment of HCAL.
Allows an energy-dependent correction for single pions which interact in ECAL.

Note that while the passive weighting can be applied to single particles and jets, the dynamic weighting may introduce high-energy tails in the case of jets

H1 weighting method @ cell level

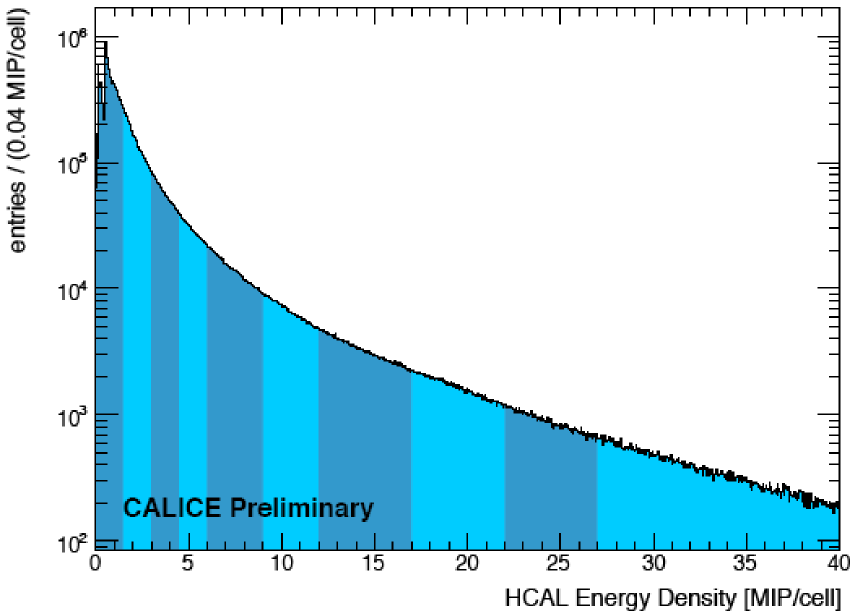
$$E'_{cell} = w\, E_{cell}$$
$$w = [c_1 \exp(-c_2\, E_{cell}/V_{cell}) + c_3]$$

- reconstruct "3D"-cluster
- split the cluster around cells with high energy density
  ➔ to separate electromagnetic from purely hadronic deposits
- apply cluster-energy and region (granularity, sub-calorimeter) dependent weights found in test beam as function of E_{cell}/V_{cell}
- tested (so far) on single particle test beam data and MC only

  should be possible to extend the method to jets :-)

  drives the need for cluster classification of the split clusters

Energy weighting @ cell level (ILC)



Energy density per detector cell in the AHCAL for 20 GeV pions

The density is calculated relative to the cell volume

The subdivision of the energy density into different bins is illustrated by the colour shading.

After accounting for different samplings and dead material reconstruct the total energy with energy dependent weightings for each *i*-cell:

$$E_{total} = \sum_i E_i \omega_i$$

Suitable weights to minimize the energy resolution are found by minimization of the χ^2 function:

$$\chi^2 = \sum_{events} \left(\sum_i E_i \omega_i - E_{beam} \right)^2$$

Energy weighting @ cell level (ILC)

$$E_{total} = \sum_i E_i \omega_i(E)$$

The weights are normally energy dependent

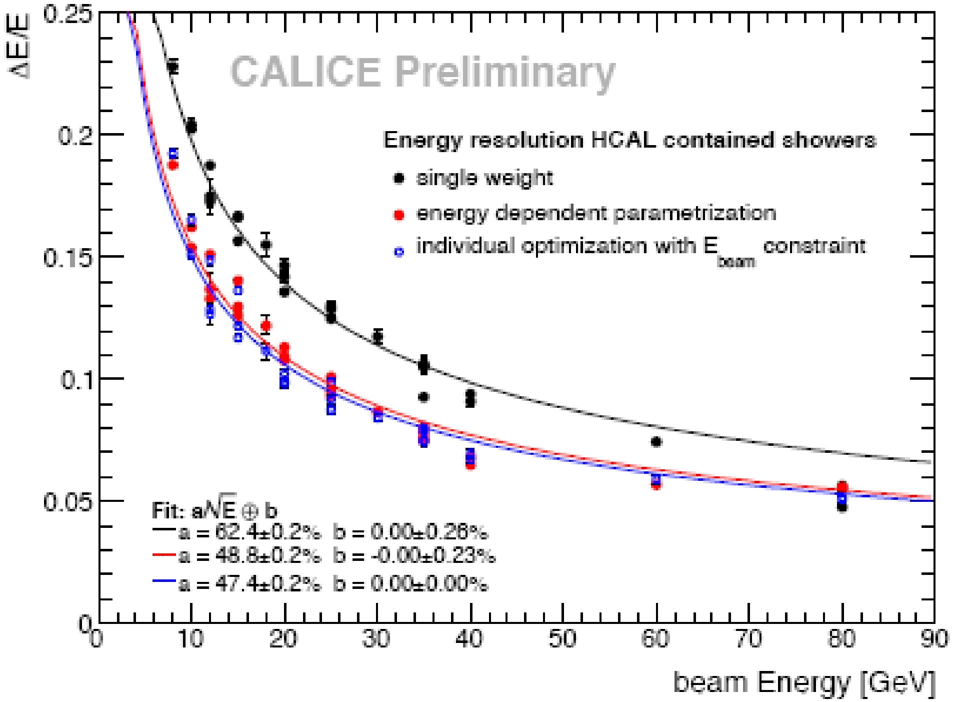
➔Requires test beam data to determine
or validated MC (possible for EM, but difficult for hadronic)

Works best when the energy on a shower is shared over many cells
➔ role of high granularity !

Once weights are applied ➔ check linearity of calorimeter response !!!

Improvements in resolution can also come from non linear behavior

Energy resolution improvement with weights from 62%/√E to 48%/√E for high granularity CALICE HCAL



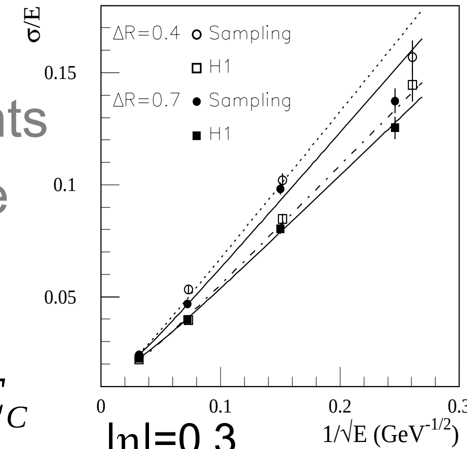
Energy weighting for Jets

Sampling Method

Back-to-back dijet events

$$E_{jet} = \alpha E_{PS} + \beta E_{EM} + \gamma E_{HAD} + \delta\sqrt{E_{EM3} \times E_{HAD}}$$

- Weights applied to different calorimeter compartments
- Enlarged cone size yields increased electronic noise



|η|=0.3

H1 Method

$$E_{jet} = E_{PS} + \sum_j \alpha_{EM}(\varepsilon_{EM,j}) \times \varepsilon_{EM,j} + \sum_j \alpha_{HAD}(\varepsilon_{HAD,j}) \times \varepsilon_{HAD,j} + \alpha_C E_C$$

- Weights applied directly to cell energies
- Better resolution and residual nonlinearities

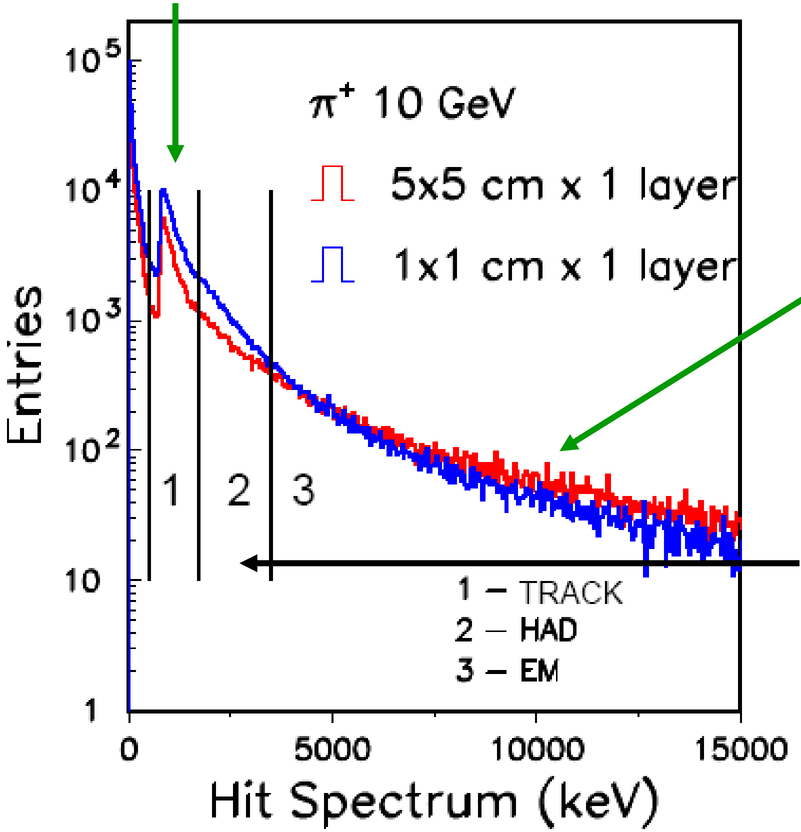
ATLAS

Parameter	Sampling Method		H1 Method	
	ΔR=0.4	ΔR=0.7	ΔR=0.4	ΔR=0.7
a (%GeV$^{1/2}$)	66.0 ± 1.5	61.2 ± 1.3	53.9 ± 1.3	51.5 ± 1.1
b (%)	1.2 ± 0.3	1.4 ± 0.2	1.3 ± 0.2	2.5 ± 0.2
χ^2 prob. (%)	1.6	0.8	27.3	66.7

Cell energy w. & topological clustering

Have a look at the hit energy spectrum per calorimeter cell

MIP-like energy deposition



Large cell energy (> 4-5 MIP) typical of EM dense cores
Remember: e/mip≠1

High granularity required!!!

Medium energy (2 – 4 MIP)
Typical of hadronic activity
~ 2-4 hadron tracks in a small calorimeter cell (~5x5 cm^2)
Remember: e/π≠1

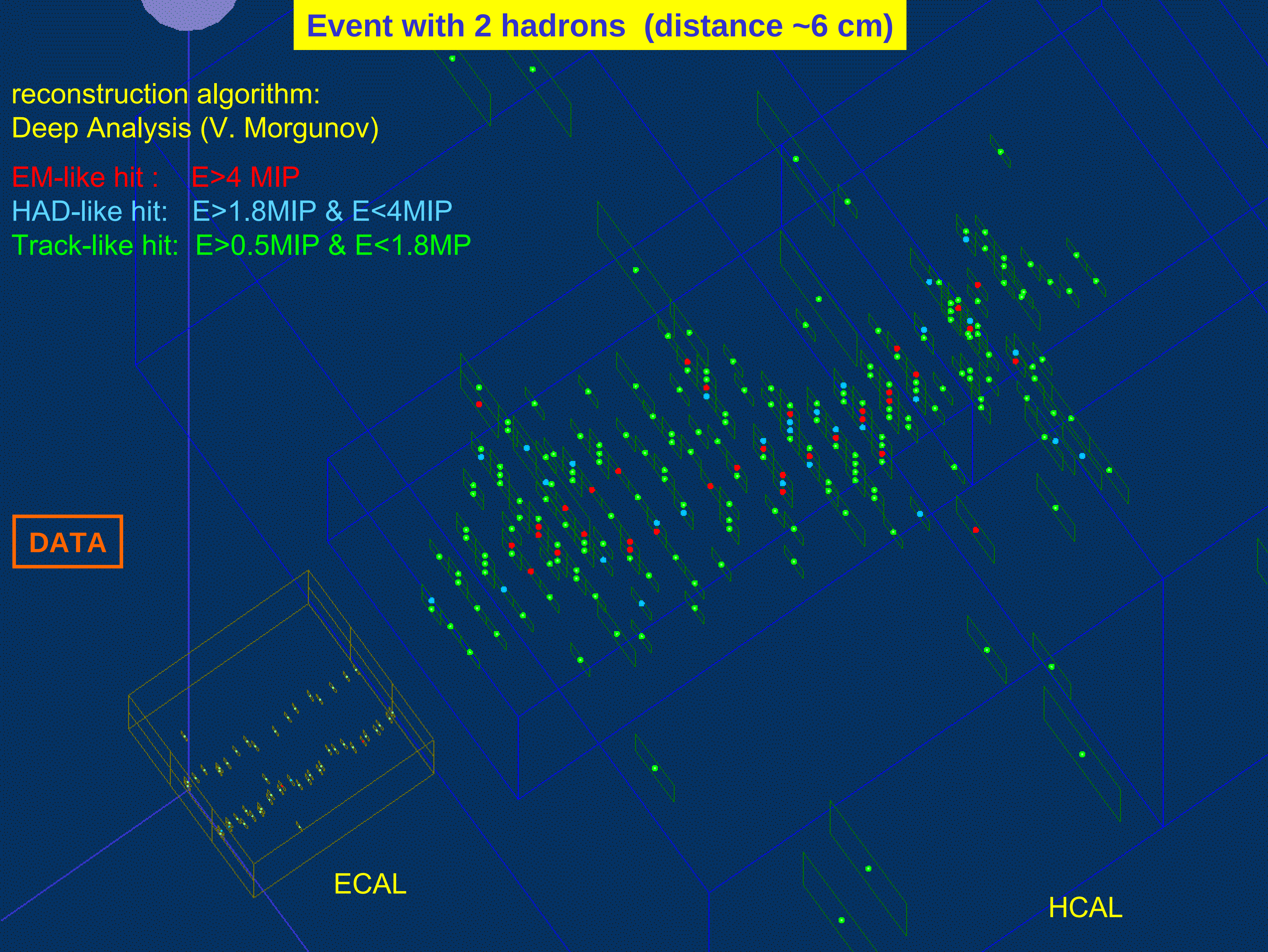
➔ use energy density as a seed for topological clustering

➔ apply E-dependent weights at cluster level according to cluster topology

Event with 2 hadrons (distance ~6 cm)

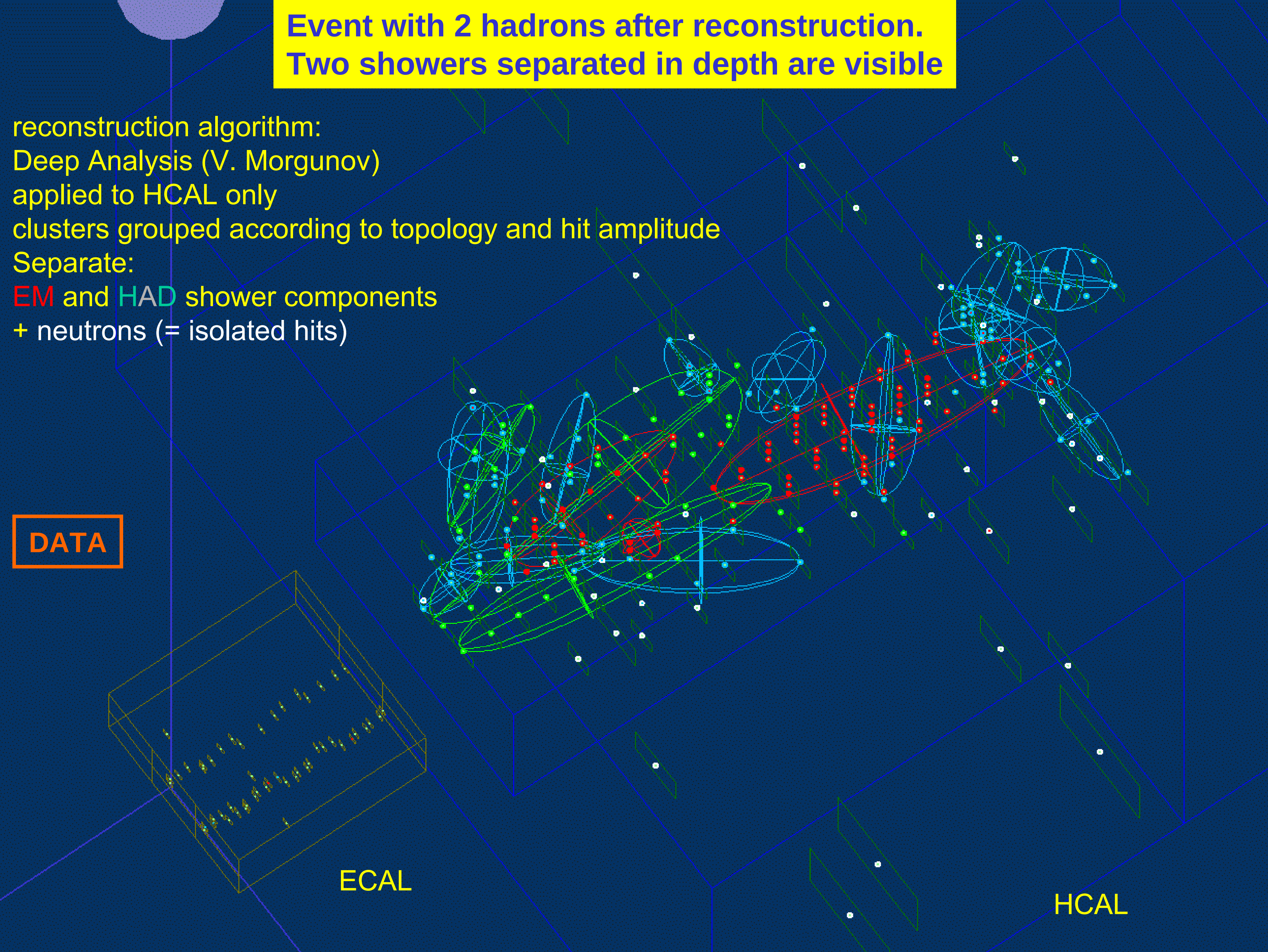
reconstruction algorithm:
Deep Analysis (V. Morgunov)

EM-like hit : E>4 MIP
HAD-like hit: E>1.8MIP & E<4MIP
Track-like hit: E>0.5MIP & E<1.8MP



Event with 2 hadrons after reconstruction. Two showers separated in depth are visible

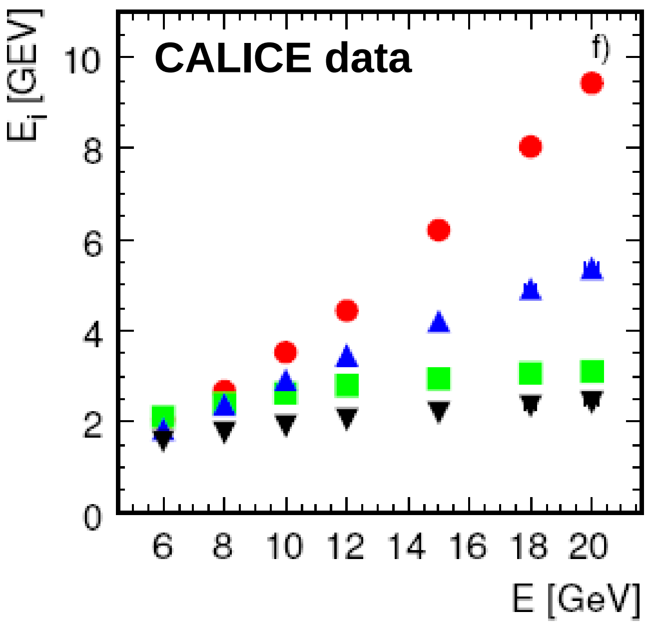
reconstruction algorithm:
Deep Analysis (V. Morgunov)
applied to HCAL only
clusters grouped according to topology and hit amplitude
Separate:
EM and HAD shower components
+ neutrons (= isolated hits)



Topological clustering

- Extremely powerful:
  identify event-by-event the EM core of single showers (EM fraction)
  hadronic and MIP-like components
- Relies on high granularity:
  to provide 3D shower density information
  allow separation of adjacent showers (in jets)

Separate shower components in:
EM–like, hadron-like, MIP-like, neutron-like ➔
directly from data without MC info



Conclusions on weighting schemes

- Weighting for different sampling structure mandatory to obtain linear response
- Energy density weighting technique applied to hadronic showers or jets improve energy resolution
- High granularity allows more accurate procedure:
  topological clustering
  more accurate weighting

Acknowledgments

These slides are largely based on Richard Wigmans lectures on calorimetry.

"Many thanks Richard for allowing me to use your material"

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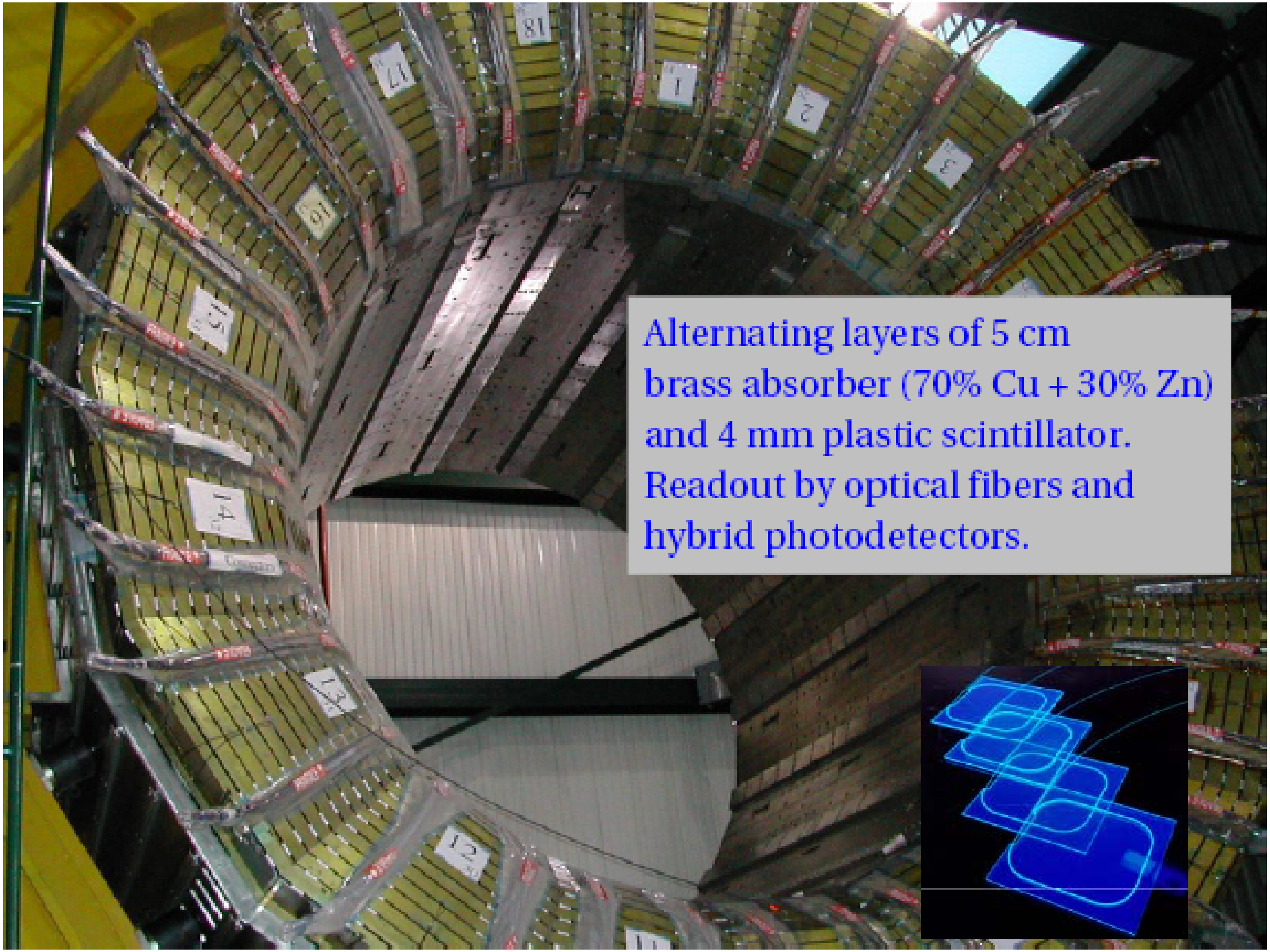
Felix Sefkow (DESY),
From whom I have taken many plots and figures

CMS Hadron calorimeter



Alternating layers of 5 cm brass absorber (70% Cu + 30% Zn) and 4 mm plastic scintillator.
Readout by optical fibers and hybrid photodetectors.

D. Pitzl, DESY

DESY summer students lecture 6.8.2008

ATLAS tile calorimeter

ATLAS LAr + Tile for pions: $$\frac{\sigma(E)}{E} = \frac{42\ \%}{\sqrt{E}} \oplus 2\ \%$$

D. Pitzl, DESY

DESY summer students lecture 6.8.2008